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MATHEMATICS FOR FIELD ARTILLERY

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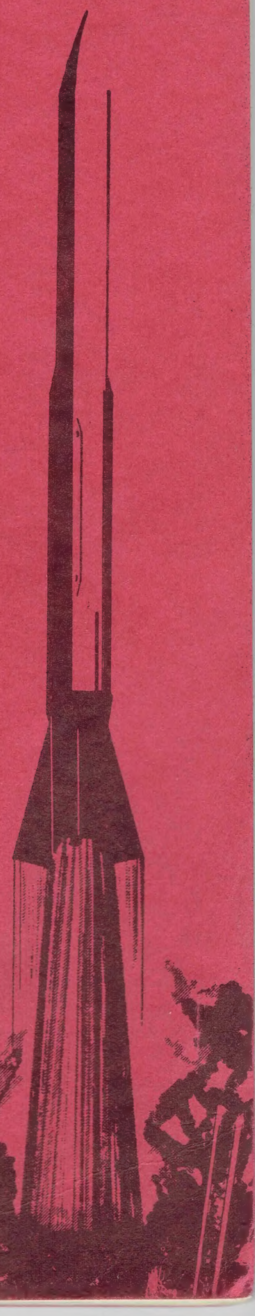
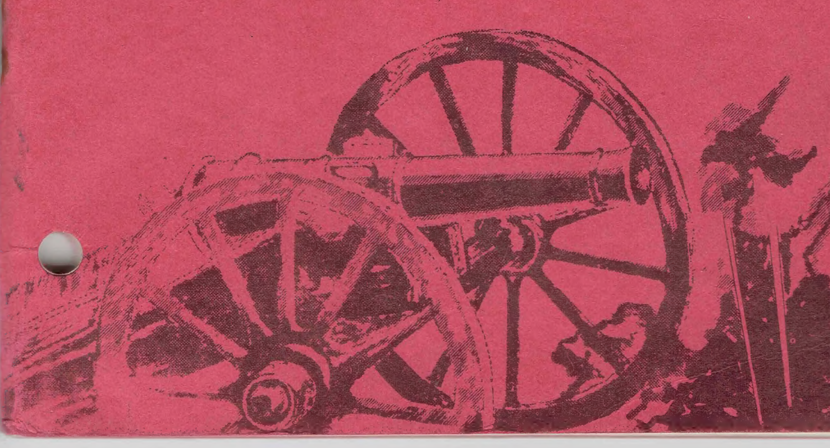
United States Army
Artillery & Missile School





PURPOSE AND SCOPE

This instructional text will provide a review of basic mathematics for field artillery. Field artillery gunnery and survey computations can be made without difficulty if the instructions herein are applied. This text does not provide a complete coverage of mathematics.

CONTENTS

Section I. ARITHMETIC

1. FRACTIONS

a. Simple fractions are a method of showing a part of a unit. The fraction 3/4 may be pictured as shown in figure 1.

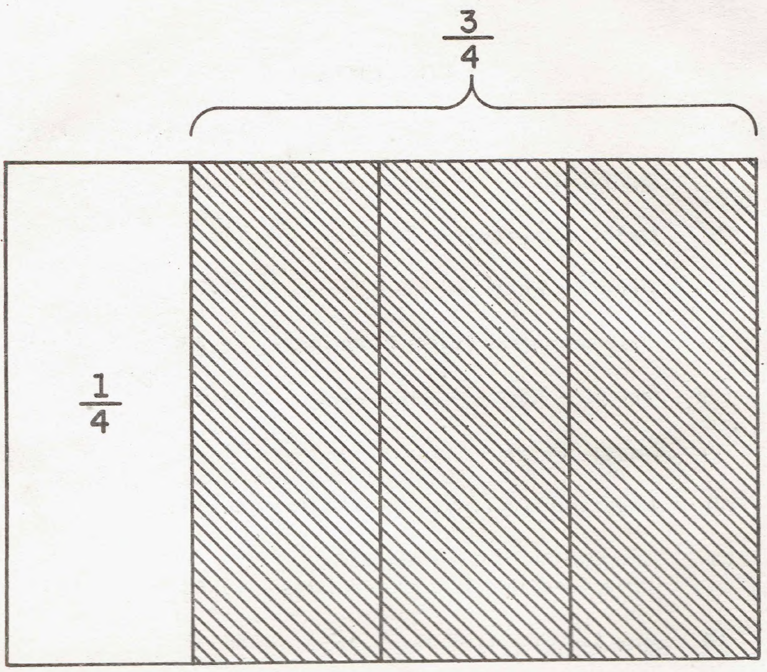


Figure 1. The fraction.

b. The terms of a fraction are the numerator, which is written above the line, and the denominator, which is written below the line. In the fraction 3/4, the 4 is the denominator and tells the number of equal parts into which the whole has been divided. The 3 is the numerator and tells the number of equal parts used.

c. A fraction in which the numerator is smaller than the denominator is called a proper fraction.

d. Fractions such as 3/3, 4/3, and 5/2 in which the numerator is equal to or greater than the denominator are called improper fractions.

e. Numbers such as 2 1/2 and 5 2/3 are called mixed numbers.

2. ADDING OR SUBTRACTING FRACTIONS OF A LIKE DENOMINATOR

a. To add one fraction to another when the denominators are the same, add the numerator of one to the numerator of the other, and place the result over the common denominator. For example, 3/12 + 4/12 = 7/12.

b. To subtract one fraction from another; find the difference between the numerators, and place it over the common denominator. For example, 8/9 - 4/9 = 4/9.

3. ADDING OR SUBTRACTING FRACTIONS OF UNLIKE DENOMINATORS

a. The value of a fraction does not change if both the numerator and denominator are multiplied or divided by the same quantity. Thus, 2/3 = 4/6 = 8/12 or 2/3 = 6/9 = 18/27.

b. If the fractions to be added or subtracted have unlike denominators, one or more of the fractions must be changed in form (but not in value) until all the denominators are alike. The fractions may then be added or subtracted in accordance with the method given in paragraph 2.

Example: 3/8 + 4/5 + 3/4 = 15/40 + 32/40 + 30/40

= 77/40

= 1 37/40

40 (the smallest number divisible by 8, 5, and 4) is the least common denominator.

4. SUBTRACTING MIXED NUMBERS

a. When the minuend (the quantity from which another is subtracted) contains a fraction larger than in the subtrahend (the quantity being subtracted) subtraction is normal.

b. When the subtrahend contains a fraction larger than in the minuend, the whole number of the minuend must be decreased by one and this one must be converted into a fraction and then added to the minuend fraction.

Thus, $2\ 1/3 - 2/3 =$

$$
\begin{array}{l}
1\ 3/3 + 1/3 \\
\underline{-\qquad\quad 2/3} \\
1\ 4/3 \\
\underline{-\ 2/3} \\
1\ 2/3
\end{array}
$$

5. MULTIPLYING AND DIVIDING FRACTIONS

a. To multiply a fraction by a whole number, multiply the numerator by the whole number. The product has the same denominator as the original fraction.

Example: $2 \times 1/4 = 2/4$

b. To multiply one fraction by another fraction, multiply the numerators, and then multiply the denominators.

Thus, $3/5 \times 4/7 = \dfrac{3 \times 4}{5 \times 7}$

$= \dfrac{12}{35}.$

c. When the product of 2 fractions, or a whole number and a fraction, is 1, one fraction is said to be the reciprocal of the other or the reciprocal of the whole number. Thus, 3/5 is the reciprocal of 5/3 and 5/3 is the reciprocal of 3/5. Likewise, 1/15 is the reciprocal of 15, etc.

d. To divide by a fraction, multiply by its reciprocal, or, simply, invert the fraction and multiply.

Thus, $5/8 \div 3/4 = 5/8 \times 4/3$

$= 20/24$

$= 5/6.$

6. DECIMAL FRACTIONS

a. General. A decimal fraction is a proper fraction in which the denominator is some power of 10 (that is, 100, 1,000, 10,000, etc.), usually signified by a point (decimal point) placed at the

left of the numerator as 2/10 = 0.2. Digits to the right of the decimal point are called tenths, hundredths, thousandths, and so on. For example, 61 27/100 = 61.27 (called sixty-one and twenty-seven hundredths). For the purpose of this discussion, the term "decimal" will be used to mean any number which includes a decimal fraction; i.e., 6.3, 0.3, 0.03, etc.

b. <u>Addition and Subtraction of Decimals</u>. In adding or subtracting decimals, write decimal point under decimal point, tenths under tenths, hundredths under hundredths, and so on.

c. <u>Multiplication of Decimals</u>. In multiplying decimals, multiply exactly as with whole numbers. Then point off as many decimal places in the product--counting from the right--as there are in the multiplier and multiplicand together. Thus, .257 x .5 = .1285. In multiplying by a decimal, it is helpful in determining placement of the decimal point to make an estimate of the product before multiplying. Thus, in multiplying 12.9 by 7.3, the whole number of the product will be nearly the same as 7 x 13. or 91. The correct product is 94.17.

d. <u>Dividing a Decimal by a Whole Number</u>. In dividing a decimal by a whole number, the same process as applied with whole numbers is used. That is, when the divisor is a whole number, the quotient always has as many decimal places as there are in the dividend; thus, 7.5÷ 5 = 1.5 or 6.325÷ 25 = .253.

e. <u>Dividing a Decimal by a Decimal.</u> In dividing a decimal by a decimal, multiply both the divisor and the dividend by an amount necessary to make a whole number of the divisor. This step may be described as moving the decimal points in both the divisor and the dividend the same number of places to the right. Then proceed with the division as in d above. For example, to divide--

82.818 by 6.42:	8281.8 by 6.42:
$6.42\overline{)82.818}$	$6.42\overline{)8281.8-}$

```
       12.9                 1290.
642 )8281.8          642 )828180.
     642
     1861
     1284
      5778
      5778
```

7. THE ROUND-OFF RULE

When a number has more digits than required, it will be rounded off to the nearest required digit. If the digits beyond the required digit have a value of less than 1/2, all digits after the last required digit are dropped. If the digits after the last required digit have a value of more than 1/2, the last required digit is increased by 1. If the digits after the last required digit have a value of exactly 1/2, the last required digit is rounded off to the nearest even number.

Example: (Round off to the nearest hundredth.)

2.76499, or 2.76 499/1000, round off to 2.76
2.76501, or 2.76 501/1000, round off to 2.77
2.76500, or 2.76 500/1000, round off to 2.76
2.77500, or 2.77 500/1000, round off to 2.78

8. THE SIMPLE PROPORTION (RATIO)

a. Comparison between two numbers may be made by use of a fraction or ratio; that is, by a ratio indicating what part one number is of another, or by division showing the number of times one number contains another. When two quantities of the same kind are compared by the ratio method, it is necessary to compare each of the quantities in terms of the same unit. For example, the ratio of 18 to 5 is 18/5 or 3 3/5. The ratio of 2 feet to 300 yards is 2/900 or 1/450 (not 2/300). The ratio of 6 inches to 6,000 feet is 6/72000 or 1/12000 (not 6/6000 or 1/1000).

b. A proportion is an expression of equality between 2 ratios, such as 8:4::6:3 (read 8 is to 4 as 6 is to 3). However, this is usually written $\frac{8}{4} = \frac{6}{3}$ which amounts to a simple equation. A proportion containing an unknown is solved by setting up an equation which represents the relationship of various values; that is, by cross multiplying and then dividing.

Thus,

(1)

$$\frac{X}{10} = \frac{6}{30}$$

$$X = \frac{6 \times 10}{30} \text{ (transposing 10)}$$

$$X = 2$$

(2)

$$\frac{6}{39} = \frac{2}{X}$$

$$6X = 2 \times 39 \text{ (transposing X and 39)}$$

$$X = \frac{2 \times 39}{6} \text{ (transposing 6)}$$

$$X = 13$$

9. SQUARING A NUMBER

To square a number is to obtain the product of two equal numbers or quantities. Hence, the square of 2 is 4; the square of 3 is 9.

10. EXERCISES

See paragraph 60 for solution.

a. Add the following fractions:

(1)	(2)	(3)	(4)
10 2/3	3 2/5	20 3/4	1 3/4
12 1/6	7 4/5	18 5/8	9 2/3

b. Subtract the following fractions:

(1)	(2)	(3)	(4)
14 5/8	15 2/3	16	21 3/4
3 1/4	10 5/6	2 1/4	6 9/10

c. Multiply the following fractions:

(1) 4/5 x 7/16 (2) 1 2/3 x 4 3/8 (3) 10 1/5 x 1/2

(4) 21 x 3 1/7

d. Divide the following fractions:

(1) 5/6 ÷ 1/2 (2) 6 1/2 ÷ 2 1/3 (3) 1 1/4 ÷ 5

(4) 44 ÷ 3 1/7

e. Multiply the following decimals:

(1)	(2)	(3)	(4)
6.459	50	520	785
.8	.2	.06	.04

f. Divide the following decimals:

(1) 6 ÷ .2 (2) 7.5 ÷ 15 (3) 6.325 ÷ 25

(4) 18.72 ÷ 1.2

g. Express the following fractions decimally giving answers to the nearest thousandth:

(1) 9/14 (2) 23/29 (3) 10 3/8 (4) 31 4/7

h. Square the following numbers:

(1) 10 (2) 24 (3) 3/4 (4) 1.2

Section II. BASIC ALGEBRA

11. ALGEBRAIC QUANTITIES

Algebraic quantities or terms have either a positive or negative value. Positive quantities are preceded either by a plus sign (+) or no sign at all. Negative quantities are always preceded by a minus sign (-). Unless the initial quantity is negative, the first term of an algebraic expression will usually have no sign.

12. ALGEBRAIC ADDITION

a. To add numbers or quantities of like signs, add the values and prefix the common sign to the result. For example--

(1) $5 + 7 + 8 = 20$.

(2) $(-8) + (-6) + (-4) = -18$.

b. To add numbers of unlike signs, determine the difference (algebraic sum) between the total values of + and - quantities and prefix the sign of the larger value to the result. For example--

(1) $17 + 5 + (-16) + 3 = 25 + (-16) = 9$.

(2) $43 + (-57) + (-13) = -27$.

(3) $6X + (-14X) = -8X$.

13. ALGEBRAIC SUBTRACTION

To subtract a quantity or term from another, change the sign of the subtrahend and perform addition as in paragraph 12b. Subtraction is merely changing every sign in the expression to be subtracted and adding it to the other expression. For example--

(1) $7 - (+5) = 7 + (-5) = 2$.

(2) $15 - (-9) = 15 + 9 = 24$.

(3) $-17 - (-4) = -17 + 4 = -13$.

14. ALGEBRAIC MULTIPLICATION AND DIVISION

The product or quotient of two quantities or numbers is positive if the signs of the quantities or numbers are alike. Otherwise, the product or quotient is negative. For example--

a. In multiplication--

$$(-5) \times (-4) = +20$$

$$5 \times (-4) = -20$$

b. In division--

$$\frac{-8}{-4} = +2$$

$$\frac{-8}{4} = -2$$

15. SIMPLE LINEAR EQUATIONS

a. An equation is formed when one term or expression is indicated as being equal to another term or expression. The quantity or expression on the right side of the equality sign is the right member or right side of the equation while that quantity or expression on the left side of the equality sign is the left member or left side of the equation. For example, in the equation $X + 5 = 10$, the left member is $X + 5$.

b. The equality of an equation is not changed if both members are treated alike. This operation may involve adding, subtracting, dividing, or multiplying both members of the equation by the same quantity.

c. A simpler procedure for performing any of the operations in b above is by transposition, in which the term or quantity is moved from one side of the equation to the other by giving it an opposing function. In transposing terms, addition and subtraction are opposing functions. In transposing functions of a member of an equation, division and multiplication are opposing functions.

Examples:

(1)	(2)
$X + 5 = 10$	$\frac{X}{5} = 10$
$X = 10 - 5$ (transpose and subtract)	$X = 10 \times 5$ (transpose and multiply)
$X = 5$	$X = 50$

d. To check the solution of any equation, substitute the solved value for the unknown in the original equation. The results should be the numerical equality.

6. EXERCISE

See paragraph 61 for solutions.

a. Solve the following equations:

(1) $4 + 15 - 19 + 7 =$

(2) $17 + 91 - 115 + 7 =$

(3) $(6) \times (-10) + \frac{18}{3}\,((6 \times 4) - 7) - \frac{(-21)}{-7} =$

(4) $(3 \times 2) + (4 \times 7) - \frac{6}{3}\,(-2) - (4 \times 27) =$

b. Solve the following equations for X:

(1) $2X + 17 = 35$

(2) $2X + 16Y = 170$

(3) $\frac{4.1}{22635} = \frac{1}{X}$

(4) $\frac{2X}{7} = 170$

(5) $\frac{3X}{5} + \frac{2Y}{7} = 21$

(6) $\frac{4.5}{X} = \frac{1}{36159}$

(7) $\frac{3.9}{45678} = \frac{X}{6532}$

(8) $\frac{2X + 5}{9} = \frac{6X - 2}{4}$

Section III. PLANE GEOMETRY

17. GENERAL

a. Plane geometry is the study of figures in a plane; a geometric figure is any combination of lines and points.

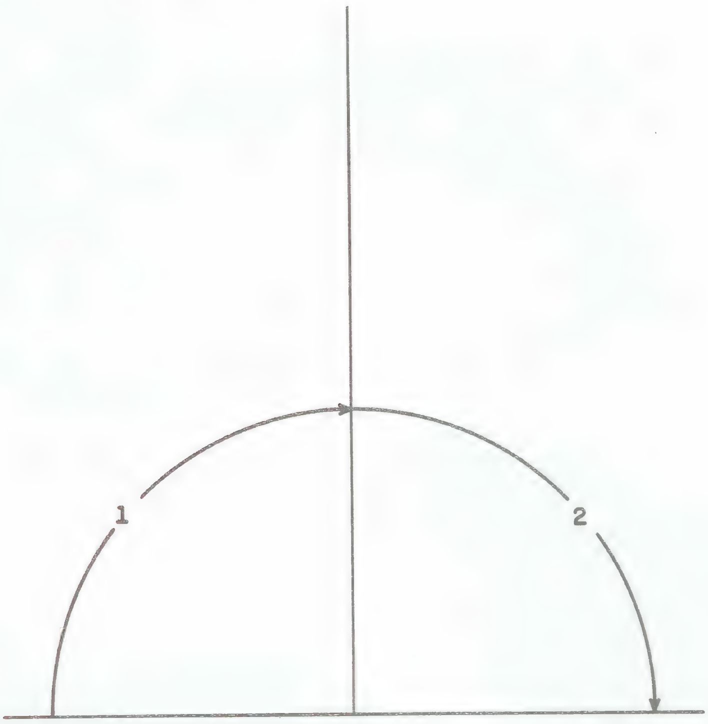
b. An angle ($\angle$) is a figure formed by two straight lines meeting at a point. The point is called the vertex, and the lines are called the sides. The length of the sides does not control the size of the angle.

c. Lines are said to be perpendicular ($\perp$) to each other when one line meets another and adjacent angles are equal. In figure 2, angle 1 equals angle 2.

18. ANGLES

a. The sides of an angle, whose vertex is at the center of a circle, determine the size of the angle by cutting a portion of the circle's circumference, creating an arc. This arc measures a certain number of mils or degrees, minutes and seconds. For

example, in figure 3, angle 1 = 1,700m̸; angle 2 = 2,400m̸; angle 3 = 2,300m̸; angle 4 = 4,000m̸; and angle 5 = 3,200m̸. The **circumference of the circle is 6,400 mils or 360°.**



<u>Figure 2. Lines perpendicular to each other.</u>

<u>b</u>. A right angle (∟) is an angle measuring 1,600 (90°) mils or the angle formed when 1 line is drawn perpendicular to another line.

<u>c</u>. An obtuse angle(⦦) is an angle measuring more than 1,600 mils (90°) but less than 3,200 mils (180°).

<u>d</u>. An acute angle (∠) is an angle measuring less than 1,600 mils (90°).

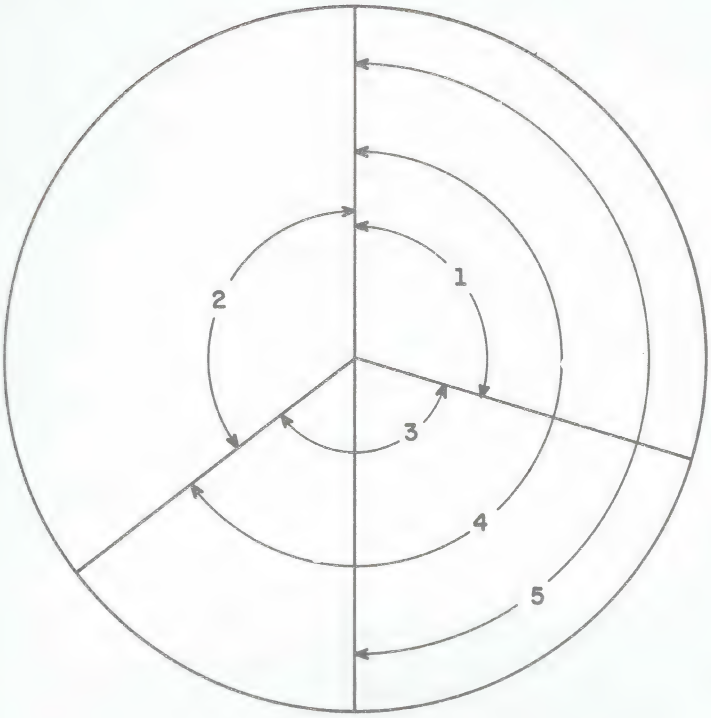


Figure 3. Angles in a circle.

e. When the sum of 2 angles equals 1,600 mils (90°) the angles are said to be complementary.

f. When the sum of 2 angles equals 3,200 mils (180°) the angles are said to be supplementary.

g. An angle of 3,200 mils (180°) is a straight line.

h. The sum of all angles around a point (fig 3) is equal to 6,400 mils or 360°.

19. BASIC GEOMETRIC THEOREMS

a. If two lines intersect, the opposite angles so formed are equal. For example, in figure 4, angle 1 is equal to angle 2, and angle 3 is equal to angle 4.

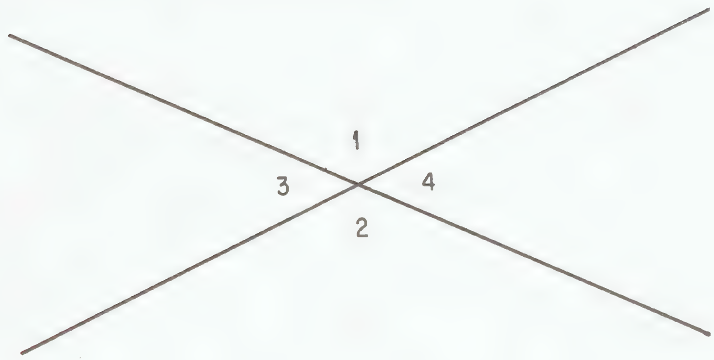


Figure 4. Intersecting lines.

b. If two parallel lines are cut by a third line (transversal), the alternate interior angles and the alternate exterior angles are equal (this rule is the basis for reciprocal laying). In figure 5, angle 4 equals angle 6 and angle 3 equals angle 5; these are the alternate interior angles. Also, angle 2 equals angle 8 and angle 1 equals angle 7; these are the alternate exterior angles.

20. TRIANGLES

a. A triangle is a figure bounded by 3 straight lines forming 3 interior angles.

b. A right triangle is a triangle in which 1 of the 3 angles is a right angle (1,600 mils or 90°).

c. An acute triangle is a triangle in which each of the 3 angles is less than 1,600 mils (90°).

d. An obtuse triangle is a triangle in which 1 of the 3 angles is greater than 1,600 mils (90°).

e. An isosceles triangle is a triangle which has two equal sides.

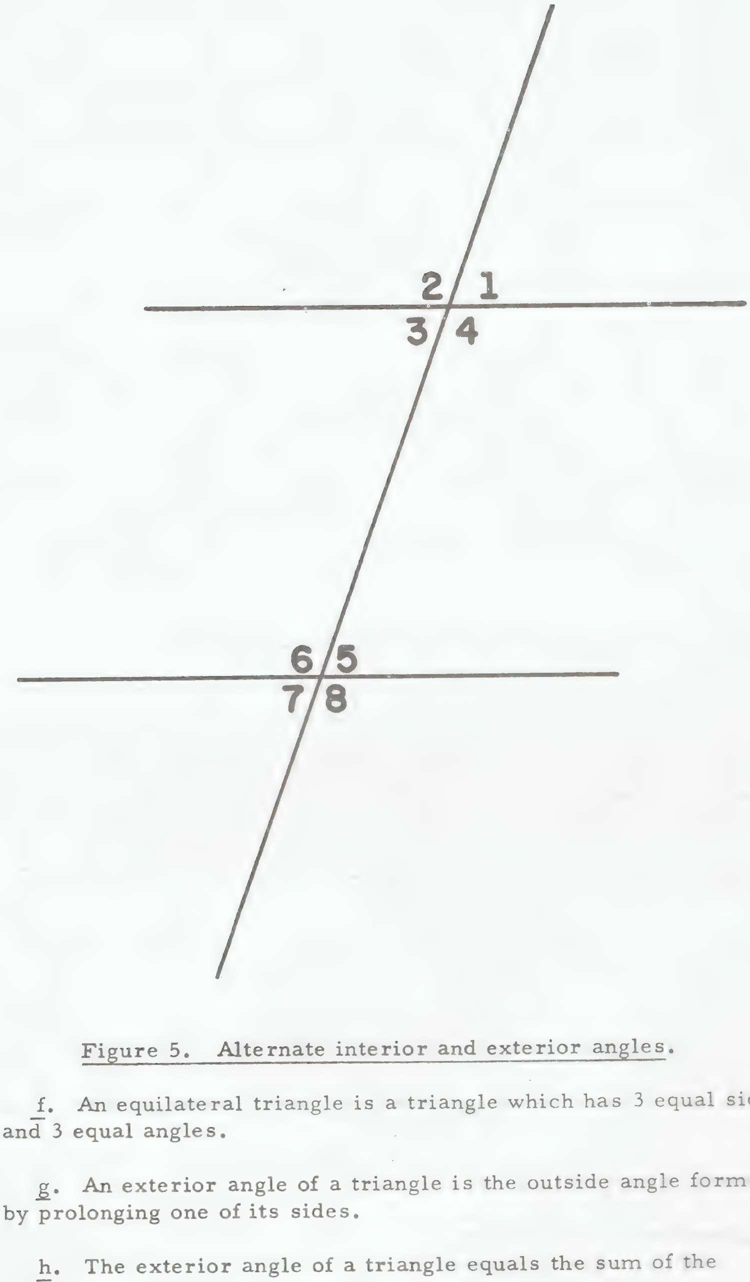


Figure 5. Alternate interior and exterior angles.

f. An equilateral triangle is a triangle which has 3 equal sides and 3 equal angles.

g. An exterior angle of a triangle is the outside angle formed by prolonging one of its sides.

h. The exterior angle of a triangle equals the sum of the opposite two interior angles (fig 6).

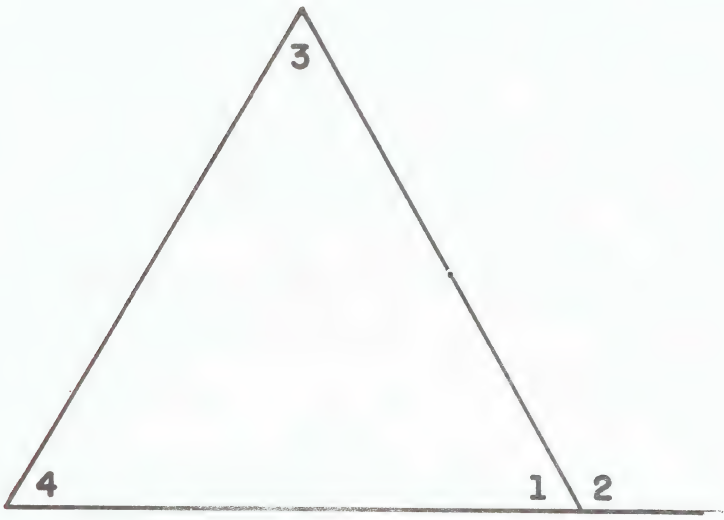


Figure 6. Exterior angle of a triangle.

i. The sum of the angles in a triangle equals 3,200 mils (180°). For proof of this statement, see figure 7. The line AB is parallel to line CD, angle 1 + angle 4 + angle 5 = 3,200 mils (180°). Angle 4 equals angle 2 and angle 5 equals angle 3 (these are the alternate interior angles). Therefore, angle 1 + angle 2 + angle 3 = 3,200 mils (180°).

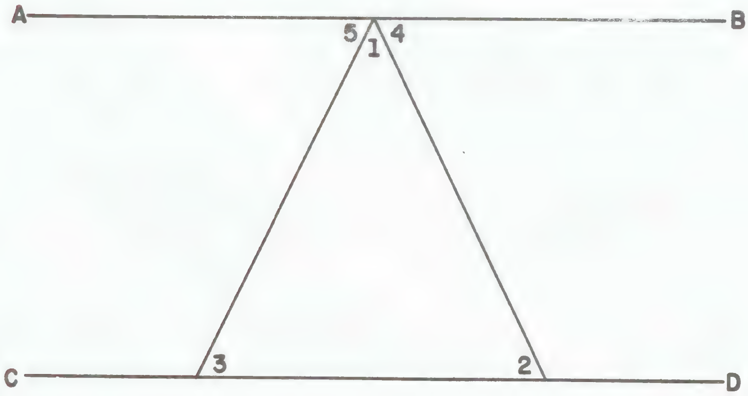


Figure 7. Interior angles of a triangle.

21. EQUA

Two tri

a. Two
sides and t

b. Thre
triangle.

c. One
and adjacer

22. SIMIL

a. Two

(1) T
o

(2) T
t

(3) A
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t

b. If a l
parallel to t
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ciple that c
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Therefore,
triangles ar
the other.

c. The e
a line is dra
be stated as

$\frac{AD}{AB} = \frac{AE}{AC}$

21. EQUAL TRIANGLES

Two triangles are equal (congruent) if--

a. Two sides and the included angle of one triangle equal two sides and the included angle of the other triangle.

b. Three sides of one triangle equal three sides of the other triangle.

c. One side and adjacent angles of one triangle equal one side and adjacent angles of the other triangle.

22. SIMILAR TRIANGLES

a. Two triangles are similar if--

(1) Three angles of one triangle are equal to three angles of the other triangle.

(2) Three sides of one triangle are proportional to the three corresponding sides of the other triangle.

(3) An angle of one triangle equals an angle of the other triangle and the respective adjacent sides are proportional.

b. If a line is drawn intersecting two sides of a triangle and parallel to the third side, the triangle thus formed is similar to the original triangle. In figure 8, angle ABC is given, and the line DE is drawn parallel to the line BC. By applying the principle that corresponding angles formed by a transversal cutting 2 parallel lines are equal, angle 2 equals angle 4, angle 3 equals angle 5, and, of course, angle 1 equals angle 1 by identity. Therefore, triangle ADE is similar to triangle ABC because 2 triangles are similar if the 3 angles of 1 equal the 3 angles of the other.

c. The example in b above illustrates the principle that when a line is drawn in a triangle as in figure 8, certain ratios may be stated as follows:

$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$$ (derived from similar triangles formed).

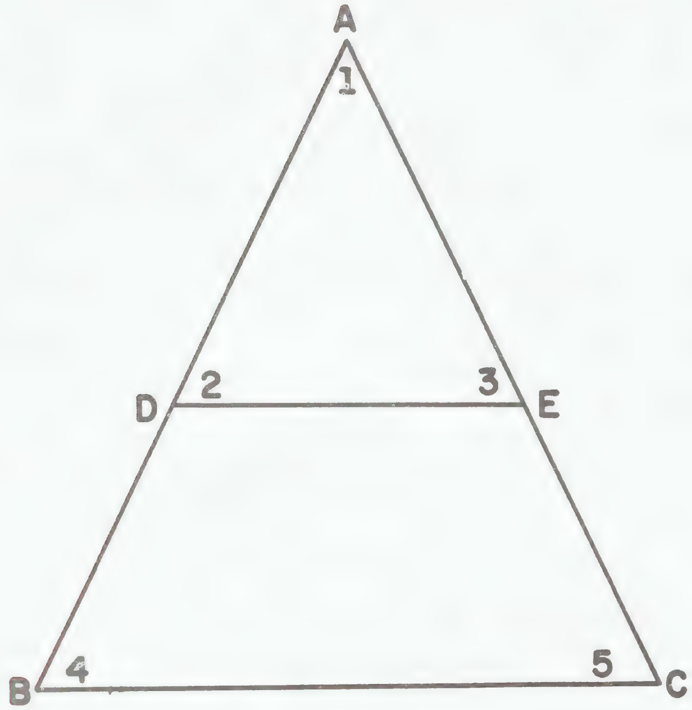


Figure 8. Similar triangles.

23. POLYGON

A polygon is a plane figure bounded by straight lines called its sides. The sum of all angles of a polygon equals (N-2) times 3,200 mils, when N equals number of sides of the polygon. This can be proved by drawing lines as shown in figure 9. The number of triangles thus formed in a polygon is always two less than the numbers of sides. The sum of the angles of a triangle equals 3,200 mils. Therefore, the sum of the angles of a polygon equals (N-2) x 3,200m.

24. PYTHAGOREAN THEOREM

a. The Pythagorean theorem states the square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides (fig 10).

$$c^2 = a^2 + b^2$$

b. **The Pythagorean theorem is used for checking the accuracy of survey computations.**

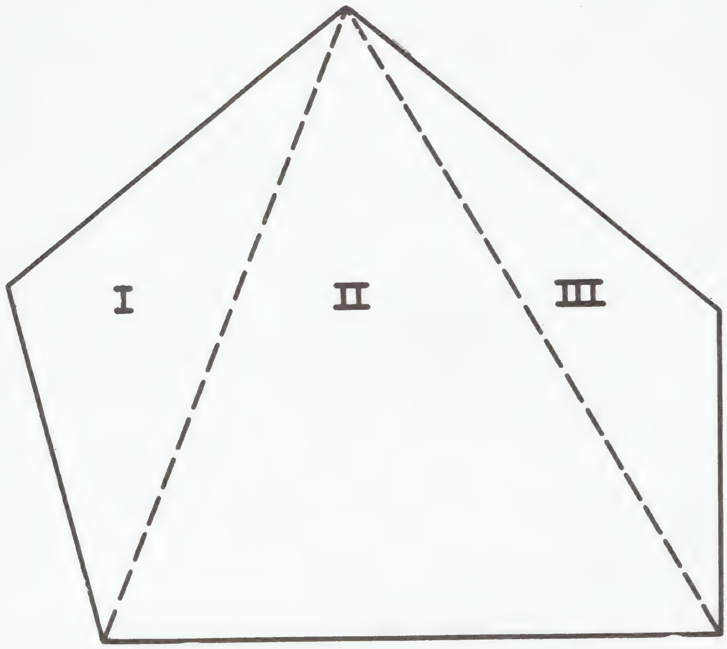


Figure 9. The polygon.

c. Example:

In figure 10, given b = 3, a = 4, what is the length of c ?

$c^2 = a^2 + b^2 = 4^2 + 3^2 = (4 \times 4) + (3 + 3) = 25$

$c^2 = 25$

Therefore, $c = \sqrt{25} = 5$

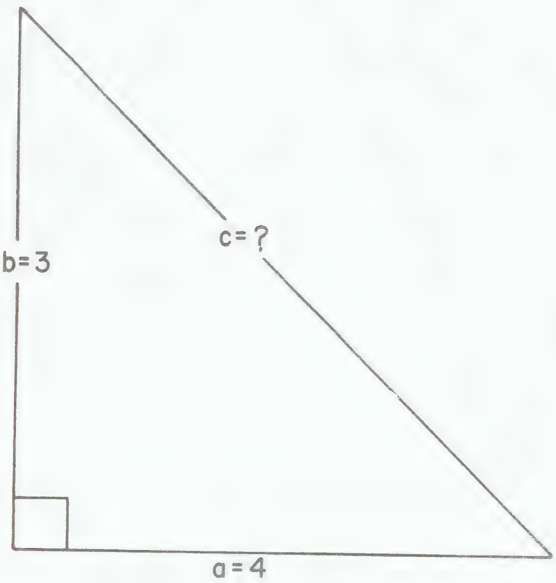
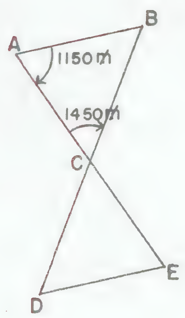


Figure 10. Pythagorean theorem.



25. EXERCISES

See paragraph 62 for solutions.

a. Given: AB ∥ DE, angle BAC = 1,150m̸, and angle ACB = 1,450m̸. Solve for--

(1) Angle B (ABC).

(2) Angle D (CDE).

(3) Angle E (CED).

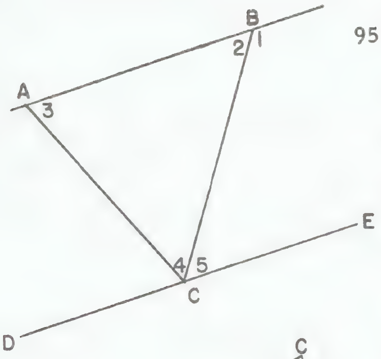
for solutions.

DE, angle
angle ACB =
--

(ABC).

(CDE).

(CED).



b. Given: AB = BC, angle 2 = 950₥, and AB ∥ DE.

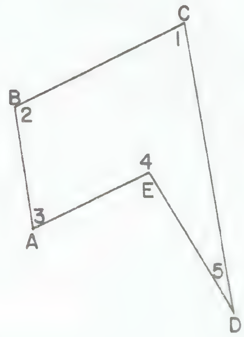
Solve for--

(1) Angle 1 = ?

(2) Angle 3 = ?

(3) Angle 4 = ?

(4) Angle 5 = ?



c. Given in polygon ABCDE--

angle 1 = 1700₥

angle 2 = 1915₥

angle 3 = 1320₥

angle 4 = 4300₥

Solve for angle 5.

Section IV. TRIGONOMETRIC (TRIG) FUNCTIONS

26. GENERAL

a. Trigonometry is a study of triangles and involves the use of trigonometric functions. A trigonometric function is the ratio between two of the sides of a right triangle. These trigonometric functions are directly relative to the size of the angles of the right triangle. Such functions are termed "natural" trigonometric functions to distinguish them from logarithms of trigonometric functions.

b. A right triangle (or right-angled triangle) is a triangle that has 1 right angle (1,600m̸). The other two interior angles are acute angles (i.e., smaller than right angles). The longest side, opposite the right angle, is called the hypotenuse. The other two shorter sides, opposite the acute angles, are called the opposite and adjacent sides (fig 11).

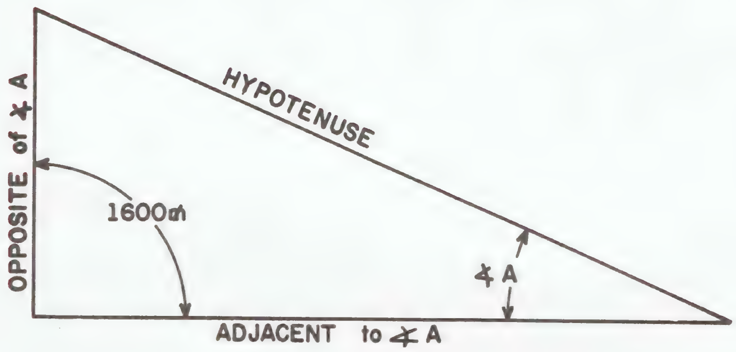


Figure 11. Sides of right triangle.

27. TRIG FUNCTIONS

The four trigonometric functions commonly used in field artillery are (fig 12)--

a. Sine--abbreviated to read sin.

$$\text{The sine of angle A} = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{a}{c}.$$

b. Cosine--abbreviated to read cos.

The cosine of angle A = $\frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{b}{c}$.

c. Tangent--abbreviated to read tan.

The tangent of angle A = $\frac{\text{side opposite}}{\text{side adjacent}} = \frac{a}{b}$.

d. Cotangent--abbreviated to read cot.

The cotangent of angle A = $\frac{\text{side adjacent}}{\text{side opposite}} = \frac{b}{a}$.

Note. By knowing the various trigonometric functions, such as--

Sin A = $\frac{a}{c}$ Tan A = $\frac{a}{b}$

Cos A = $\frac{b}{c}$ Cot A = $\frac{b}{a}$

It is possible to find the unknown in any of the above equations if the other two values are known. For instance, if angle A and side a are known, then b = $\frac{a}{\tan A}$ and c = $\frac{a}{\sin A}$. Likewise, if side c and angle A or B are known, it is possible to find sides a and b. The sum of the interior angles of any triangle is 3,200 mils (180°).

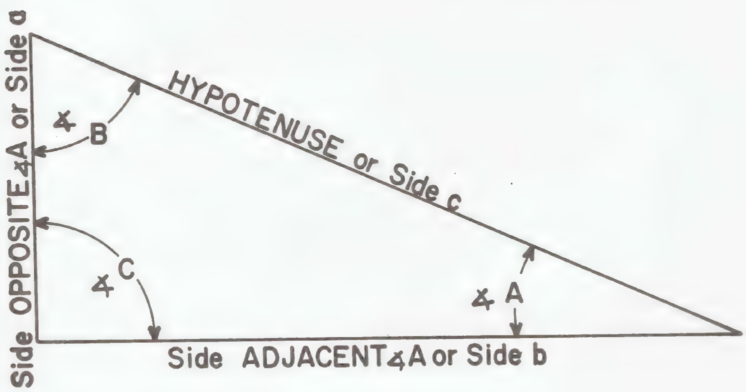


Figure 12. Trigonometric functions

28. THE UNIT CIRCLE

a. The unit circle is used to illustrate functions of angles. The unit circle (fig 13) is divided into four quadrants--first (I), second (II), third (III), and fourth (IV). The radius of the circle is considered to be unity or one. Considering the angle at O (angle QOP), the sine is $\frac{QP}{OP} = \frac{QP}{1}$ or the distance QP; similarly, the cosine is the distance OQ, and the tangent is the distance Q'P'. From figure 13, when angle O is 0, the cosine is equal to 1 and the sine and tangent QP and Q'P', respectively, are 0. As the angle at O increases to 1,600₥, the cosine decreases to 0 and the sine becomes 1 or unity. The tangent is infinity insomuch as the tangent line is parallel to the line QP.

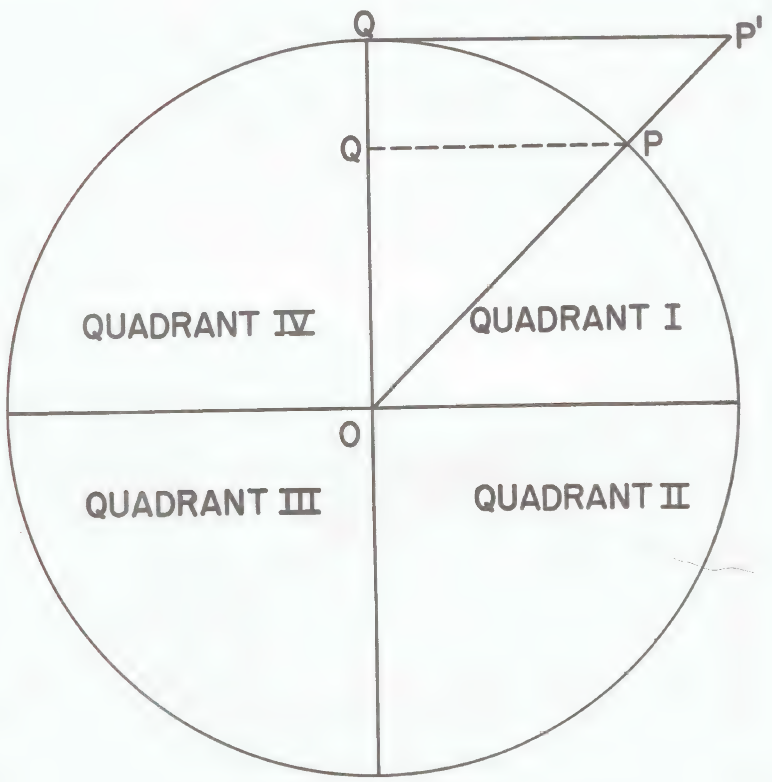


Figure 13. Unit circle.

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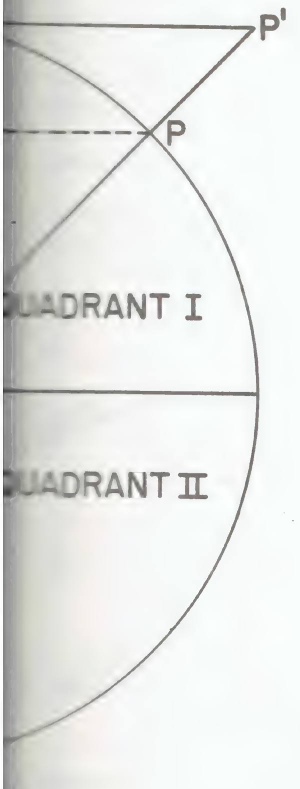

b. In figure 14, the angle shown is 800 mils (45°) in quadrant I, 2,400 mils (135°) in quadrant II, 4,000 mils (225°) in quadrant III, and 5,600 mils (315°) in quadrant IV. In quadrants I and II, the sine is to the right of the origin or plus in sign; in quadrants III and IV, the sine is to the left of the origin or minus in sign. The cosine is plus in quadrants I and IV and minus in quadrants II and III.

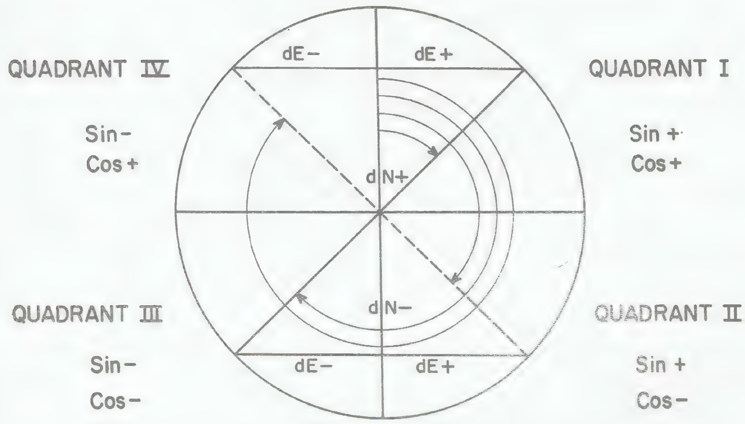

Figure 14. Signs of the four quadrants.

29. THE BEARING ANGLE

a. The bearing angle is the acute angle formed when an azimuth or line intersects the north-south line (vertical line in fig 14). The numerical value of the bearing angle is determined as follows:

bearing angle quadrant I = az

bearing angle quadrant II = 3,200 (180) - az

bearing angle quadrant III = az - 3,200 (180)

bearing angle quadrant IV = 6,400 (360) - az

b. Since the angle is measured from north or south toward east or west, this numerical value is prefixed by north or

south and suffixed by east or west. In the first quadrant, the bearing angle is north (so much) east (N so much E), in the second quadrant, S so much E, in the third quadrant, S so much W, and in the fourth quadrant, N so much W. When determining this angle, it is always best to draw a figure

30. APPLICATION OF FUNCTIONS TO FIELD ARTILLERY SURVEY

a. The grid azimuth (az) of a line is the basic direction for field artillery survey. It is the horizontal, clockwise angle between grid north and a given line. In figure 15, the azimuth of the line AB converted to a bearing angle represents one interior angle of a right triangle.

b. If the azimuth and distance (dist) of the line AB are measured, dE (easting) can be computed by using the formula: Sin of the bearing angle = $\frac{-dE}{dist}$. dN (northing) can be computed by using the formula: Cos of the bearing angle = $\frac{-dN}{dist}$.

c. If the coordinates of point A and point B are known, the azimuth of the line AB can be computed by using the formula: Tan of the bearing angle = $\frac{dE}{dN}$. After computing the bearing angle, the azimuth can be determined through the algebraic signs of the dE and dN which place the angle in the appropriate quadrant. The distance of the line AB can be computed by using 1 of 2 formulas--if dE is larger, the formula in (1) below should be used; if dN is larger, the formula in (2) below should be used.

(1) Sin of the bearing angle = $\frac{dE}{dist}$.

(2) Cos of the bearing angle = $\frac{dN}{dist}$.

31. SINE AND TANGENT FUNCTION AND ARCS OF SMALL ANGLES

a. In small angles, the sine, the arc, and the tangent are approximately equal. As the angle increases, this approximate equality no longer exists, and the tangent increases more rapidly than the sine. Figure 16 illustrates this principle. The radius of the circle being 1 unit, the sin P = $\frac{DE}{OE} = \frac{DE}{1}$ = DE and

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ARCS OF SMALL

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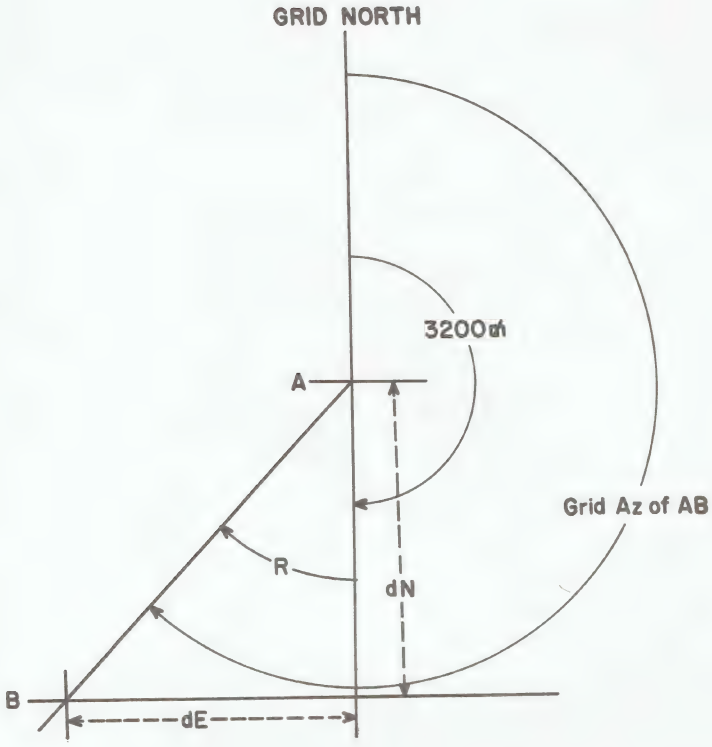


Figure 15. Computation of grid azimuth.

$\tan P = \frac{D'E'}{OD'} = \frac{D'E'}{1} = D'E'$. Likewise, sin M and tan M are represented graphically by BC and B'C', respectively. (Note that this is merely graphical representation. Sines and tangents are ratios, not lines.)

b. In the small angle M, the sine (BC), the arc (B'C), and the tangent (B'C') are approximately equal, while in the larger angle P, the arc (D'E) is appreciably greater than the sine (DE), and the tangent (D'E') is considerably greater than either. (Note that this disparity increases as the angle increases.)

c. Figure 16 shows the arc to be intermediate between the graphically represented sine and tangent. It is thus evident that for small angles the ratio of arc over radius may be

considered as equal to the tangent without introducing large errors. (Note that a 6-place table of natural functions (TM 6-230) gives the sin of $40\not{m}$ to be 0.03926 and the tan of $40\not{m}$ to be 0.03929. This illustrates the close agreement between sine and tangent when the angle is small.)

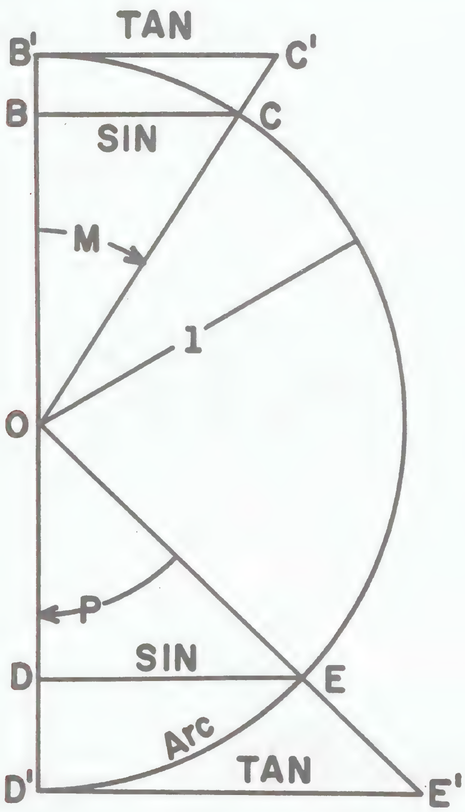


Figure 16. Functions of small angles

32. THE MIL RELATION

a. The unit of angular measurement most commonly used in artillery is the mil. A mil subtends 1/6400 of the circumference of a circle, since there are 6,400 mils to the circle just as there are 360°. Therefore, a degree contains approximately 17.8 mils.

b. Most field artillery angle-measuring instruments (except the transit and theodolite) are graduated in mils; for example, binoculars, aiming circle, panoramic telescope, BC scope, and gunner's quadrant.

c. The mil relation is based on the tangent function of a right triangle and is used for the approximate solution of certain triangles. It is expressed as a simple equation: $\not{m} = \frac{W}{R}$, where $\not{m}$ = mils, W = width in yards or meters, and R = range or distance in thousands of yards or meters. By substitution in this formula, an angle of 1 mil ($\not{m}$) will subtend a width (W) of 1 yard at a distance (R) of 1,000 yards; i.e., $\not{m}$ (1) = $\frac{W \text{ (1 yard)}}{R \text{ (1/1000)}}$. If any two factors of the formula are known, the third factor may be determined (fig 17). If this approximate mil unit is applied to a circle, the sum of individual units will equal 6,283 mils instead of 6,400 mils. For small angles, the inaccuracy caused by the approximate mil unit is negligible; for angles of 400 to 600 mils, only a rough approximation is obtained; and for angles greater than 600 mils, the mil relation should not be used (para 30c). There are several ways to remember the mil relation rule; one way is the WORM rule: W over RM. Cover the desired unknown and the remaining portion is the equation.

Example: $\frac{W}{RM}$. The value of W is known to be 40 yards and R to be 8,000 yards. By covering the letter M, the equation $\frac{W}{R}$ is derived; therefore, 40 divided by 8 (8,000 yards in thousands of yards) is equal to 5 $\not{m}$ (fig 17).

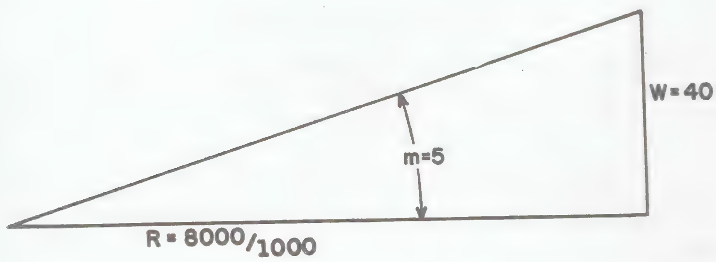
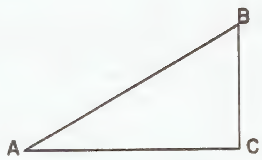


Figure 17. Use of WORM rule.

33. EXERCISES

See paragraph 63 for solutions.



a. In the right triangle shown, label the sides from angle A.

b. Complete the following trigonometric functions:

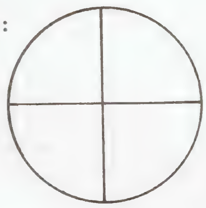
$$\text{Sin } A = \frac{\qquad\qquad}{\text{hypotenuse}}$$

$$\text{Cos } A = \frac{\qquad\qquad}{\text{hypotenuse}}$$

$$\text{Tan } A = \frac{\text{side opposite}}{\qquad\qquad}$$

$$\text{Cot } A = \frac{\qquad\qquad}{\text{side opposite}}$$

c. Label the four quadrants of unit circle:



d. Solve the following, using the mil relation rule:

Angle, 20 mils; distance, 1,000 meters; width =

Distance, 3,000 meters; width, 30 meters, angle =

Angle, 5 mils; width, 20 meters; distance =

Section V. THE SINE LAW

34. THE SINE LAW (SINE PROPORTIONAL)

The sine law is often used by field artillery personnel to compute distance to an unknown point which may be inaccessible; for example, a point in the target area.

a. In any triangle, the ratio of any side to the sine of the angle opposite that side is equal to the ratio of any other side to the sine of the angle opposite that other side. The law of sines is--

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

b. For proof of this law, a perpendicular line (h) has been drawn from C (fig 18).

(1) Sin $A = \frac{h}{b}$ or $b \sin A = h$.

(2) Sin $B = \frac{h}{a}$ or $a \sin B = h$.

(3) b Sin $A = a \sin B$ (since they both equal h).

(4) $\frac{a}{\sin A} = \frac{b}{\sin B}$ by dividing (3) by (sin A sin B).

(5) Likewise, it can be proved that $\frac{a}{\sin A} = \frac{c}{\sin C}$.

Note. **The above is true of any triangle, either right or oblique. This formula is important for it is used extensively in field artillery survey, and, frequently, it is employed by the gunnery officer in computations of a high-burst or center-of-inpact registration to obtain the length of one side of a triangle. The value of 2 angles and the length of 1 side are normally known.**

c. In figure 19, the measured length of the base (b) is 530 meters. The angles at C and A are measured by the aiming circle to be angle C = 1,548₥, **angle A = 1,452₥.** Angle B = 3,200 - (1,548 + 1,452) = 200₥. The formula $\frac{a}{\sin A} = \frac{b}{\sin B}$ is used to solve for side a. Then, $a = \frac{b \sin A}{\sin B}$, substituting one value.

$$a = \frac{b \sin A}{\sin B}$$

$$a = \frac{530 \sin 1{,}452}{\sin 200}$$

$$a = \frac{530 \times 0.98946}{0.19509}$$

a = 2,688 meters

```
   .98946                 2,688.06
      530        .19509 524.41380.00
  2968380               390 18
 494730                 134 233
524.41380               117 054
                         17 1798
                         15 6072
                          1 57260
                          1 56072
                            118800
                            117054
                               746
                               8925
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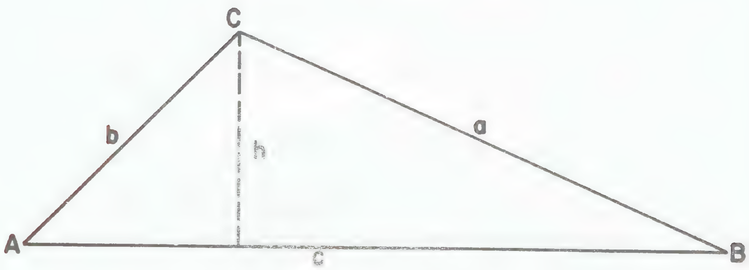
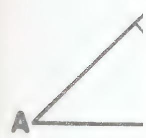


Figure 18. Ratio of sides of triangl

35. DETERMINATION OF THE INTERIOR ANGLES OF THE TARGET AREA BASE TRIANGLE

a. When the OP's are not intervisible, the interior angles of the target area base triangle for the target area survey are determined by comparing the azimuths of the sides of the triangle. The 01-02 azimuth is furnished by survey personnel. The azimuth from 01 or 02 to the target is read from the scale of the instrument (azimuth readings are always used in a high-burst or center-of-impact registration; this reading will be the average of six readings to the burst).

b. In figure 20, the apex angle equals the azimuth 02 to target (T) minus the azimuth 01 to target. The interior angle at 01 (angle 1) equals the azimuth 01 to target minus to 01-02



azimuth. T
azimuth min
left, the rel
would be rev

Example:

Azimuth (

Azimuth (

Azimuth (

Azimuth (

Apex angl

Base leng

Angle 1

Angle 2

c. To sol
located, the l

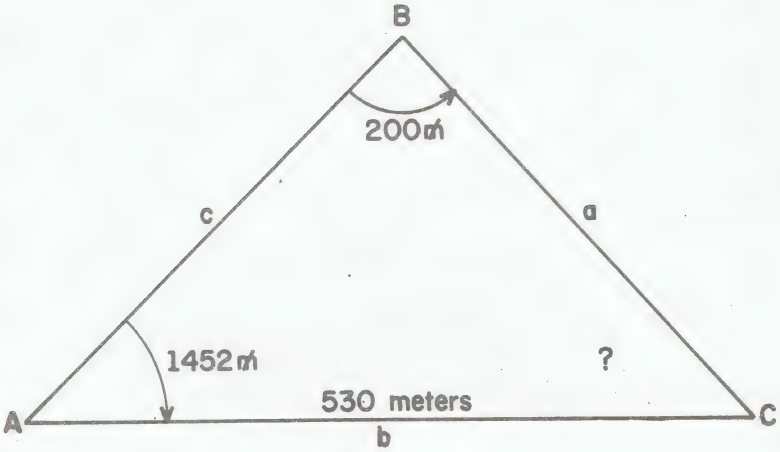


Figure 19. Computation of one side of triangle.

azimuth. The interior angle at 02 (angle 2) equals the 02-01 azimuth minus the azimuth 02 to target. If 01 were on the left, the relationship of 01 and 02 in the foregoing computation would be reversed.

<u>Example:</u>

Azimuth 01-02	=	505m̸ (furnished by survey personnel)
Azimuth 02-01	=	3,705m̸ (01-02 azimuth + 3,200 m̸)
Azimuth 01-T	=	2,097m̸ (from instrument azimuth scale)
Azimuth 02-T	=	2,346m̸ (from instrument azimuth scale)
Apex angle	=	249m̸ (2,346 - 2,097)
Base length	=	625 meters (furnished by survey personnel)
Angle 1	=	1,592m̸ (2,097 - 505)
Angle 2	=	1,359m̸ (3,705 - 2,346)

<u>c</u>. To solve for the distance 01 (02) to the point to be located, the law of sines (para 34) is used.

$$\frac{\text{Distance}}{\text{Sin opposite interior angle}} = \frac{\text{base}}{\text{sin apex angle}}$$

$$\text{Distance} = \frac{\text{base x sin opposite interior angle}}{\text{sin apex angle}}$$

To solve this equation using natural functions of angles in mils, distance = base x sin opposite interior angle ÷ sin apex angle. The military slide rule is arranged to provide a simple and rapid solution of the target area base problem (TM 6-240).

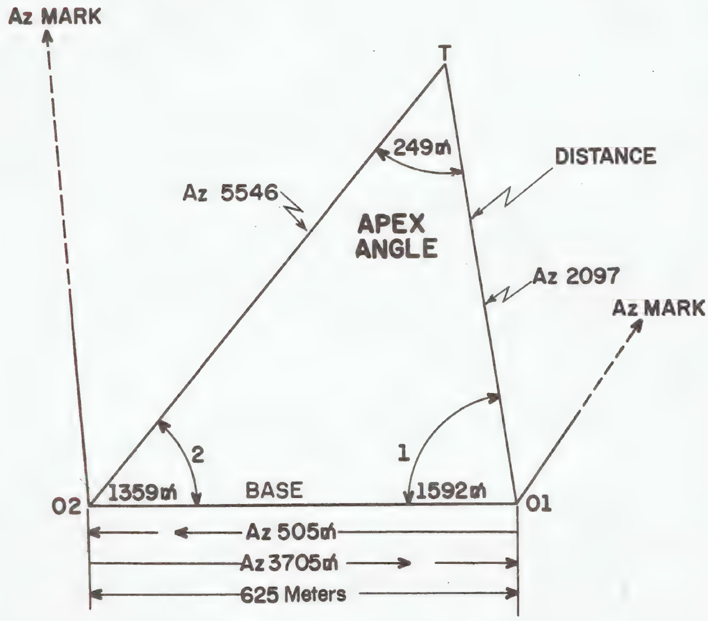


Figure 20. The target area base triangle.

36. EXERCISE

See paragraph 64 for solutions.

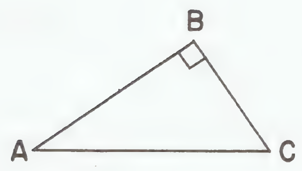
a. Length of b = 5,000 meters

Angle A = 700 mils

Angle B = 1,500 mils

Angle C = 1,000 mils

Solve for side c using natural functions of angles in mils.

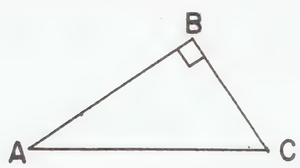


b. Azimuth of line AB = 800 mils

Azimuth of line AC = 1,600 mils

Angle ACB = 1,100 mils

Solve for azimuth of line CB.
Solve for angle B.



Section VI. INTERPOLATION

37. SINGLE INTERPOLATION

Interpolation is the procedure by which intermediate values can be determined between any 2 numbers or any 2 points in a series. Fundamentally, interpolation consists of determining the proportional part between two known values.

a. Graphic interpolation is commonly practiced in any measurement made with a scale wherein the point to be measured does not fall exactly on a graduation mark. In this case, the interpolation may be done by using a vernier or, more commonly, by estimation.

b. Mathematical interpolation in a series is basically the same as the graphical interpolation but the process is performed mathematically and is, therefore, more accurate. The fundamental problem consists of determining a proportional part between two known values. This may be accomplished either by setting up a proportion and solving for the unknown or by mental calculation.

c. For mathematical interpolation, at least 2 series of numbers bearing a relationship are needed, so arranged that corresponding numbers in any 2 series can be selected easily by the interpolator. Many examples of these series of numbers are found. Most interpolation for the field artillery is performed in firing tables and log tables, which are arranged in series. Hence, in the firing tables, elevations and times can be found for corresponding ranges; similarly, ranges can be found from elevations and times. In the log tables, a number corresponding to a certain logarithm, or a logarithm which corresponds to a certain number, can be found.

d. To save space and paper, all series cannot include all numbers within a series. Therefore, if a number in one series and its counterpart in another corresponding series do not appear in the printed tables, interpolation must be performed. Interpolation is inserting an intermediate number between 2 numbers printed in the tables; the fundamental problem then is to determine the proportional part between 2 known values. For example, it is desired to find an elevation corresponding to 4,760 in the series of ranges for charge 5 (shell HE, 105-mm howitzer). Since this exact range is not printed in the range series, interpolation is necessary. Because this range (4,760) is .60/100 or 0.6 of the distance between 4,700 and 4,800, the elevation falls 0.6 of the way between 265.0 mils (4,700) and 272.1 mils (4,800). The problem then is to determine a corresponding

elevation 0.6 of the way from 265.0 (4,700) to 272.1 (4,800). This interpolation may be done by computing 0.6 of the distance from 265.0 and 272.1 or 0.6 x (272.1 - 265.0) = 0.6 x 7.1, which is 4.26, or 4.3. Adding 4.3 to 265.0, an elevation of 269.3 is obtained which corresponds to range 4,760 (4,700 + 60). This problem may also be solved by using a proportion as shown below.

Range		Elevation	
	4700	265.0	
	60	X	
100	4760	?	7.1
	4800	272.1	

$$\frac{60}{100} = \frac{X}{7.1}$$

$$X = \frac{60}{100} \times 7.1 = 4.26$$

Elevation corresponding to 4,760 = 265.0 + 4.3 = 269.3.

e. To find a range corresponding to a given elevation (el), the same principles in d above apply. For example, it is desired to find the range corresponding to an elevation of 216.1 (105-mm how, chg 5, sh HE). The firing tables show that 216.1 falls between listed elevations 211.6 and 218.0. What is the proportional distance from 211.6 to 218.0? From 211.6 to 218.0 involves a distance or difference of 6.4; 216.1 is $\frac{216.1 - 211.6}{6.4}$ or $\frac{4.5}{6.4}$ of the way from 211.6 to 218.0. The

problem is to determine a number in the range series which is a proportional distance between 3,900 and 4,000 (range corresponding to el 211.6 and 218.0). $\frac{4.5}{6.4}$ of (4,000 - 3,900) = $\frac{4.5}{6.4}$ x 100, or 70.3. The required range is 3,900 + 70, or 3,970. Solution by a proportion is as follows:

6.4	211.6	3900	6.4 corresponds to 100
	4.5	x	4.5 corresponds to X
	216.1	? — 100	
	218.0	4000	

$$\frac{X}{100} = \frac{4.5}{6.4} \qquad X = \frac{100 \times 4.5}{6.4} = 70.3$$

Range corresponding to elevation 216.1 = 3,900 + 70.3 = 3,970.

f. Interpolation in the log tables is performed in the same manner (d and e above). For this purpose, consider the log tables as only two series of numbers with a relationship. Call one series the numbers, and the other series the logs. The examples below are used only to illustrate interpolation and are not to be considered as logarithmic problems. Logarithms as such are covered in section VII.

Example: To find log corresponding to 27654.4--

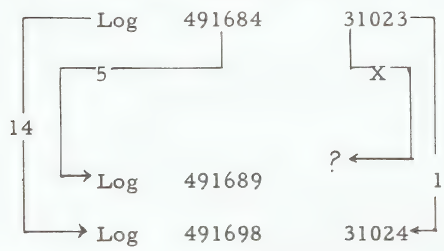
1	Nr 27654	Log 441758	1 corresponds to 16
	.4	X	.4 corresponds to X
	Nr 27654.4	? — 16	
	Nr 27655	Log 441774	

$$\frac{X}{16} = \frac{.4}{1}$$

$$X = \frac{16 \times .4}{1} = 6.4, \text{ or } 6$$

Nr 27654.4 = log 441758 + 6, or log 441764.

To find antilog of 491689--



5 corresponds to X
14 corresponds to 1

$$\frac{X}{1} = \frac{5}{14}$$

$$X = \frac{1 \times 5}{14} = .36$$

Log 491689 = Nr 31023 + .36, or 31023.36.

g. In order to save time when interpolating, use the tables of proportional parts (P. P.) which are printed on the margin of the pages of the tables of logarithms. Thus, in the first example in f above, having noted that 27654.4 will be 4/10 the difference between log 441758 and 441774 (16), refer to the marginal column headed P. P. (proportional parts) and locate the column headed 16. Under this column is found the various tenths of 16. 4/10 of 16 is given as 6; therefore,

The log of 27654 = .441758

4/10 of 16 = 6

Finally, the log 27654.4 = .441764

This is the same result obtained by interpolating mathematically above.

38. DOUBLE INTERPOLATION

Double interpolation consists of three procedures--

a. Determining the proportional part between two known values in a series of known values.

b. Using the proportional part that has been determined as a known value in a second set of known values.

c. Determining a proportional part between two known values (one of which has been determined in a above) in the second set of values.

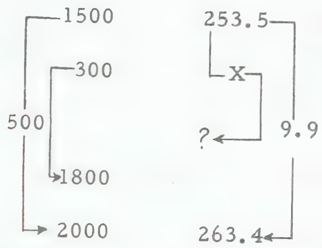
Example: (This example is used to illustrate double interpolation and is not to be considered as factual data.)

A burst 1,800 feet above a target is to be fired at a range of 17,460 yards, using charge 4 (280-mm gun, shell HE, T122). Information from firing tables:

Range in yards	Height of target above gun (feet) 1500	2000	
17400	253.5	263.4	Elevations
17500	255.4	265.2	

1st Interpolation: The firing tables give the elevations to fire with a height of burst of 1,500 feet and 2,000 feet at the ranges of 17,400 yards and 17,500 yards, respectively. Therefore, it is necessary to interpolate to determine what elevations are required to fire a height of burst of 1,800 feet at each of these ranges. This is done in two steps, as follows:

Step 1



500 corresponds to 9.9
300 corresponds to X

$$\frac{300}{500} = \frac{X}{9.9} \qquad X = \frac{300}{500} \times 9.9 = 5.94$$

$$\begin{array}{r} 253.5 \\ +\ 5.9 \\ \hline 259.4 \end{array}$$ = elevation for range 17,400 with height of burst of 1,800 feet.

Step 2

	1500	255.4
	300	X
500	1800	?
	2000	265.2

500 corresponds to 9.8
300 corresponds to X

9.8

$$\frac{300}{500} = \frac{X}{9.8} \qquad X = \frac{300}{500} \times 9.8 = 5.88$$

$$\begin{array}{r} 255.4 \\ +\ 5.9 \\ \hline 261.3 \end{array}$$

= elevation for range 17,500 with height of burst of 1,800 feet.

2d Interpolation: The elevations to fire with a height of burst 1,800 feet above the target at both ranges (17,400 and 17,500) have been determined. Now take the values determined and interpolate for the elevation to fire with a height of burst 1,800 feet above the target at a range of 17,460 yards.

	17400	259.4
	60	X
100	17460	?
	17500	261.3

100 corresponds to 1.9
60 corresponds to X

1.9

$$\frac{60}{100} = \frac{X}{1.9} \qquad X = \frac{60}{100} \times 1.9 = 1.14$$

$$\begin{array}{r} 259.4 \\ +\ 1.1 \\ \hline 260.5 \end{array}$$

= elevation for range 17,460 with height of burst of 1,800 feet.

39. EXERCISES

See paragraph 65 for solutions. Use 105-mm how, sh HE, chg 5, in exercises a, b, and c.

a. Find fuze setting for graze burst (fz M500) and elevation corresponding to each of the following ranges:

(1) 4,765.
(2) 3,941.
(3) 6,740.
(4) 4,310.

b. Find range and elevations corresponding to the following fuze settings for graze burst (fz M500):

(1) 10.8.
(2) 11.4.
(3) 17.8.

c. Find range corresponding to the following elevations:

(1) 317.3.
(2) 339.4.
(3) 368.1.

d. Find log numbers corresponding to the following numbers (TM 6-230):

(1) 57018.
(2) 23459.
(3) 45601.
(4) 11238.

e. Find numbers corresponding to the following log numbers (TM 6-230):

(1) 521203.
(2) 731041.
(3) 093562.
(4) 170291.

Section VII. LOGARITHMS

40. GENERAL

a. Logarithms are used in field artillery computations to save time and work. By application of several simple rules governing logarithms, long, tedious calculations may be avoided. The common logarithm of a number is the power to which 10 must be raised to equal that number.

Examples: Log 1000 = 3; $10^3 = 1000$

Log 100 = 2; $10^2 = 100$

Log 10 = 1; $10^1 = 10$

Log 1 = 0; $10^0 = 1$

Log .1 = -1; $10^{-1} = 1/10 = .1$

Log .01 = -2; $10^{-2} = 1/10^2 = .01$

Log .001 = -3; $10^{-3} = 1/10^3 = .001$

b. The logarithm of a number is the sum of two parts--the characteristic plus the mantissa. Logarithms are expressed as a whole number (the characteristic) followed by a decimal fraction (the mantissa). The characteristic is determined by rule and the mantissa is taken from tables of Common Logarithms of Numbers (Table I, TM 6-230 or Table I, VEGA TABLES).

Example:

		Characteristic	Mantissa
Log 4376.1	=	3	.641087

41. CHARACTERISTICS

a. To find the characteristic of a logarithm, determine the "standard position" of the decimal point. The standard position is between the first and second significant digits in any number. In the following examples the standard position is shown:

4 732.10   4 73.21   4 7321   0.4 7321   0.004 73

b. To find the characteristic of the logarithm of any number, determine how many places the decimal point must be moved to put it in the standard position. The number of places moved equals the characteristic. Thus, for the number 763.82, since the decimal point must be moved 2 places left (7 63.82), the characteristic is 2.

c. If the number to be logged is greater than 1, the characteristic has a positive sign; if less than 1, it has a negative sign.

7 6.382 Characteristic = 1 (Positive sign is not normally shown.)

7 6382.00 Characteristic = 4

0.007 6382 Characteristic = $\bar{3}$ (Negative sign is written above the characteristic to indicate that only the characteristic, not the mantissa is negative.)

0.7 6382 Characteristic = $\bar{1}$

d. For convenience, a negative characteristic is usually expressed as a positive number minus 10; that is, a characteristic of $\bar{1}$ would be expressed as 9-10. The logarithm of .05799 is $\bar{2}$.763401, and it would be expressed as 8.763401-10.

42. MANTISSA

The mantissa of the logarithm of a number is determined by reference to tables and does not depend on the position of the decimal point. The mantissa depends only on the digits of a number and is always positive. It remains unchanged as long as the sequence of digits in the number remains unchanged. For example, the mantissa for 4400, 440, 44, 4.4, .44, and 0.44 is .643453. The mantissa of 1835 or 183.5, etc., is .263636. For example--

Number	Characteristic +	Mantissa =	Logarithm
123.5	2	.091667	2.091667
0.523	$\bar{1}$ (or 9-10)	.718502	9.718502-10

43. DETERMINATION OF DECIMAL POINT

After the number corresponding to a given mantissa has been determined, the location of the decimal point in the number must be found. From the standard position, count off a number of places equal to the characteristic of the logarithm. If the characteristic has a positive sign, the number must be greater than 1; if negative, the number must be less than 1.

Logarithm	Number
4.820201	6 6100.00
1.820201	6 6.100
9.820201-10 ($\bar{1}$.820201)	.6 6100
7.820201-10 ($\bar{3}$.820201)	.006 6100

44. LOGARITHMS OF FUNCTIONS OF ANGLES

a. Since artillery computations will often involve the functions of angles (i.e., sin, cos, tan, etc.) with numbers, these computations will be simplified by the use of logarithms. The sine or cosine of all angles, except 0 and 1,600₥ (0° and 90°), are less than unity; thus the logarithm of the sine or cosine has a negative characteristic. It is, therefore, necessary to add -10 to the number taken from the tables. The same applies to the tangent of an angle less than 800₥ (45°). Logarithms of functions of angles in mils are found in table II, TM 6-230.

Example: Log sin 732.3₥ is 9.818610-10 (page 358)

Log cos 411₥ is 9.963643-10 (page 294)

Log cot 962₥ is 9.859466-10 (page 339)

Log tan 325₥ is 9.518982-10 (page 277)

b. Logarithms of functions of angles in degrees, minutes, and seconds are found in table II and III of Vega logarithmic tables or table II, TM 5-236.

Example: Log sin 49° 32' is 9.88126-10 (page 68, TM 5-236) or 9.8812612-10 (page 532, Vega tables)

45. MULTIPLICATION AND DIVISION OF NUMBERS BY LOGARITHMS

a. Since logarithms are exponents, the logarithm of a product is the sum of the logarithms of the numbers being multiplied. Thus, log 20 = log (10 x 2) = log 10 + log 2 = 1.000000 + 0.301030 = 1.301030. For division, the logarithms of the two numbers are subtracted, the divisor being subtracted from the dividend. Examples of time saved in using logarithms are as follows:

Without logs	Using logs
4.732	Log 4.732 = 0.675045
x38.54	+Log 38.54 = 1.585912
18928	2.260957
23660	
37856	
14196	Antilog of .260957 = 182372
182.37128	Characteristic of 2 gives 182.372 as result

```
      0.01755
785.6√13.787280
      7856
      59312
      54992
       43208
       39280
        39280
        39280
```

Log 13.787280 = 1.139479
-Log 785.6 = 2.895201
8.244278-10

Antilog of .244278 = 175501
Characteristic of 8-10 gives 0.01755 as result

b. To determine the length of a side of a triangle when the length of 1 side and the value of 2 angles are known, the use of logarithms simplify the computations. To solve the equation in paragraph 34, using logarithms, log distance = log base + log sin opposite interior angle - log sin apex angle. To solve the problem given in paragraph 34, using logarithms--

Log 530	2.724276
+Log sin 1452	9.995399-10
Sum	12.719675-10
-Log sin 200	9.290236-10
Difference	3.429439

Antilog of 3.429439 = 2,688.06 meters (side a)

46. RAISING A NUMBER TO A POWER

Raising a number to a power using logarithms is simply an operation of multiplying the logarithm of that number by the power. For example, the equation $100^3 = ?$ is solved by multiplying the logarithm of the number (100) by the power (3). Log 100 = 2,000000. Log 100^3 = 2.000000 = 6.000000. The antilog of the logarithm 6.000000 is 1,000,000. Thus 100^3 = 1,000,000.

47. EXTRACTING A ROOT

Extracting a root using logarithms is simply an operation of dividing the logarithm of the number by the indicator of the desired root. For example, to extract the square root of 1,000,000 ($\sqrt{1,000,000}$) it is necessary to divide the logarithm of 1,000,000 by 2. Log $\sqrt{1,000,000}$ = 6.000000÷2 = 3.000000.

The antilog of the logarithm 3.000000 is 1,000. Thus

$\sqrt{1,000,000} = 1,000$. 1,000 is the square root of 1,000,000.

48. EXERCISES

See paragraph 66 for solution.

a. Determine the following:

(1) Log sin 215m̸.

(2) Log sin 937m̸.

(3) Log cos 625m̸.

(4) Log cos 1821.8m̸.

(5) Log 132.8.

(6) Log 0.0570.

(7) Log tan 518.7m̸.

(8) Log tan 2218m̸.

(9) Log 24433.

(10) Log .000451.

(11) Antilog 8.654369-10.

(12) Antilog .621592.

(13) Antilog 6.823102.

b. By the use of logarithms, determine--

(1) 137.5 x 7.18.

(2) Sin 820m̸ x 2508.

c. By the use of logarithms, determine--

(1) 48.37÷9.612.

(2) 442÷tan 77.6m̸.

Section VIII. CALCULATION OF GRID AZIMUTH, DISTANCE, OR GRID COORDINATES

49. GENERAL

a. The procedure to calculate azimuth and distance from coordinates or coordinates from azimuth and distance is similar.

b. In calculating grid azimuths, distance, and coordinates, the results must be correct, for errors in artillery survey or firing data are likely to have very serious consequences.

50. CALCULATION OF GRID AZIMUTHS FROM GRID REFERENCES

a. The tangent function (para 27c) is used in calculation of grid azimuths from coordinates, and the process is sufficiently precise to warrant computing to the nearest tenth of a mil or to the nearest second, although this manual only computes to the nearest whole number in azimuth as well as in coordinates.

b. To determine the grid azimuth of the line AB in figure 21 when the coordinates of point A are 5864092430 and the coordinates of point B are 5648090650, first determine the difference between easting coordinates and the difference between northing coordinates, shown as dE and dN in figure 21. Next, determine the quadrant in which point B lies with respect to point A; that is, northeast (I), southeast (II), southwest (III), or northwest (IV). In this case, point B is southwest of point A because both coordinates for point B are smaller than those for point A. Therefore, the grid azimuth sought must be between 3,200 mils and 4,800 mils.

c. From figure 21, grid azimuth AB = 3,200 mils plus angle R.

(1) By using logarithms--

Point A--5864092430

-Point B--5648090650

dE = 2160

dN = 1780

$$\text{Tan } R = \frac{dE}{dN} = \frac{2160}{1780}$$

Log 2160 = 3.334454

Log 1780 = 3.250420

Log tan R = 0.084034

R = 897.9 mils

Grid azimuth AB = 3,200m̸ + 897.9m̸ = 4,097.9m̸.

(2) By using natural functions--

$$\text{Tan } R = \frac{dE}{dN} = \frac{2160}{1780} = 1.21348 = 898$$

Grid azimuth AB = 3,200m̸ + 898 = 4,098m̸.

51. CALCULATION OF DISTANCE FROM COORDINATES

The sine or cosine function (para 30) is used to determine the distance between coordinates. The comparative length of dE and dN will determine which function to use. If dE is longer, use the sine function; if dN is longer, use the cosine function. In figure 21, the dE = 2160 and the dN = 1780.

a. $\text{Sin bearing angle} = \dfrac{\text{opposite side}}{\text{hypotenuse}}$

$$\text{Distance} = \frac{dE}{\text{sin bearing}}$$

Log 2160 = 13.334454-10

Log sin 897.9 = 9.887449-10

Log distance = 3.447005

Distance = 2,799 meters.

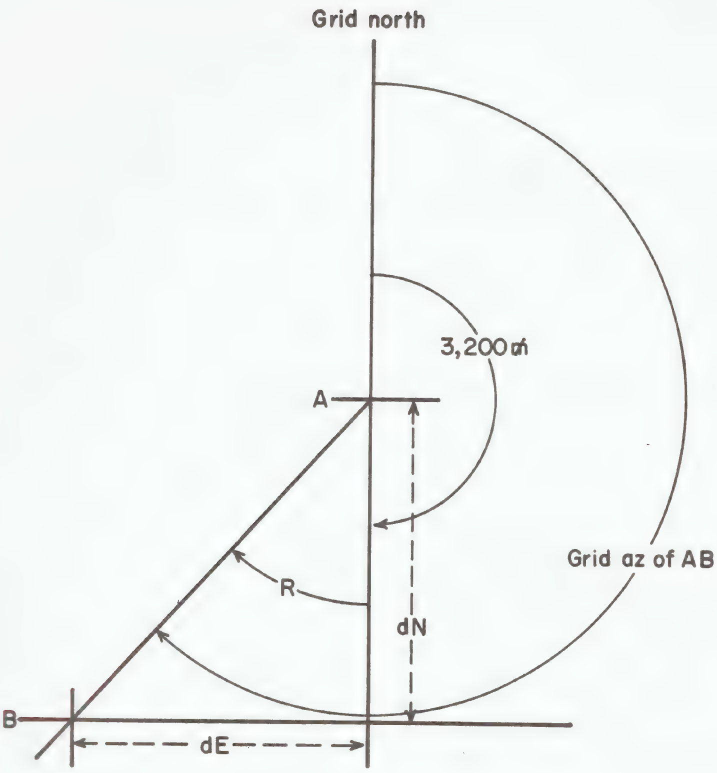


Figure 21. Computation of grid azimuth.

b. Cos bearing angle $= \dfrac{\text{adjacent side}}{\text{hypotenuse}}$

$$\text{Distance} = \frac{dN}{\cos \text{bearing}}$$

$$\text{Log } 1780 = 13.250420\text{-}10$$

$$\text{Log cos } 897.9 = \underline{9.803447\text{-}10}$$

$$\text{Log distance} = 3.446973$$

$$\text{Distance} = 2,798.8 \text{ meters.}$$

52. CALCULATION OF COORDINATES FROM GRID AZIMUTHS AND DISTANCE

To determine the coordinates of a point when the grid azimuth and distánce from, and the coordinates of, a second point are known, the sine and cosine functions (para 30) are used. From the information given in figure 22, the coordinates of point B may be determined.

<u>a</u>. <u>To find dE</u>--

$$\begin{aligned} \text{Log } 2118 &= 3.325926 \\ \text{Log sin } 985\text{m̸} &= \underline{9.915505-10} \\ \text{Log dE} &= 13.241431-10 \\ \text{dE} &= +1,743.53 \text{ meters.} \end{aligned}$$

<u>b</u>. <u>To find dN</u>--

$$\begin{aligned} \text{Log } 2118 &= 3.325926 \\ \text{Log cos } 985\text{m̸} &= \underline{9.754160-10} \\ \text{Log dN} &= 13.080086-10 \\ \text{dN} &= +1,202.50 \text{ meters.} \end{aligned}$$

<u>Note</u>. The angle 985m̸ measured from grid north makes point B fall in quadrant I; therefore, dE is plus and dN is plus.

Coordinates point A	58640	94580
dE and dN	+1743.53	+1202.50
Coordinates point B	60383.53	95782.50

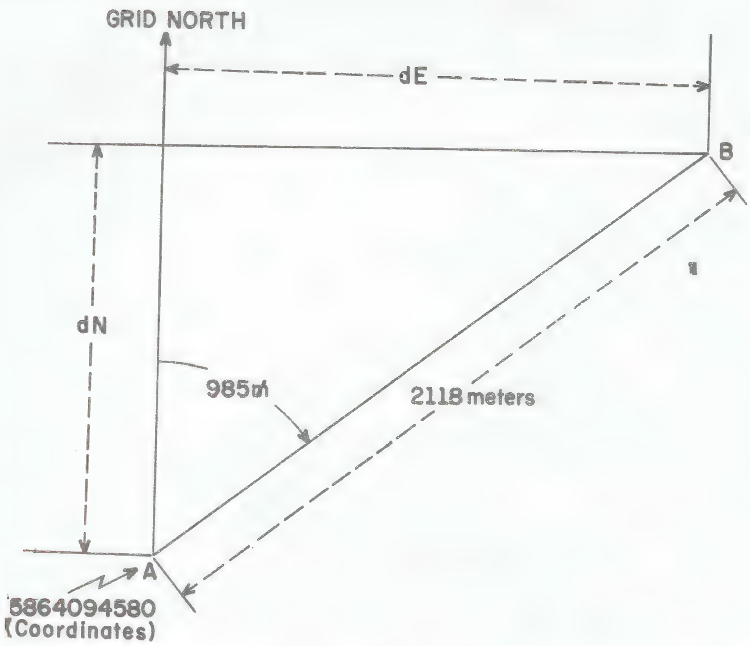
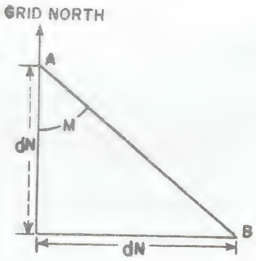


Figure 22. Computation of coordinates.

53. EXERCISES



See paragraph 67 for solutions.

a. Determination of distance.

Coordinates of point A are 5548294370.

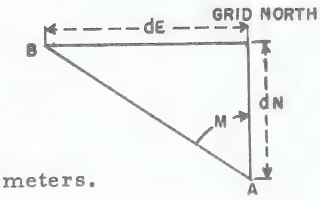
Coordinates of point B are 5807591860.

(1) The grid azimuth of line AB is ___________m̸ .

(2) The bearing angle (angle M) is ___________m̸ .

(3) The distance from A to B is ____________meters.

b. Determination of coordinates.



Coordinates of point A are 5045096640.

Angle M is 1,095₥.

Distance from A to B is 2,150 meters.

(1) The dE is __________.

(2) The dN is __________.

(3) The coordinates of point B are ___________.

Section IX. ANGULAR CONVERSIONS

54. GENERAL

Table III, TM 6-230, can be used to convert angles in degrees, minutes, and seconds to the corresponding value in mils and to convert angles in mils to the corresponding value in degrees, minutes, and seconds.

55. CONVERSION OF AZIMUTH (ANGLES) FROM MILS TO DEGREES, MINUTES, AND SECONDS

Table IIIa, page 374, TM 6-230 (conversion of mils to degrees, minutes, and seconds) contains five independent subtables.

a. The left table at the top gives values in degrees, minutes, and seconds for each 1,000 mils for angles from 1,000 mils to 6,000 mils.

b. The right table at the top gives values in degrees, minutes, and seconds for each 100 mils for angles from 100 mils to 900 mils.

c. The left table (second row) gives values in degrees, minutes, and seconds for each 10 mils for angles from 10 mils to 90 mils.

d. The right table (second row) gives values in minutes and seconds for each 1 mil for angles from 1 mil to 9 mils.

e. The last table gives values in minutes and seconds for tenths of mils for angles from 0.1 mil to 0.9 mil. An example of converting mils to degrees, minutes, and seconds (azimuth 3,622₥) follows:

0 mils	=	168° 45' 00''
600 mils	=	33° 45' 00''
20 mils	=	1° 07' 30''
2 mils	=	0° 06' 45''
3,622 mils	=	203° 44' 15''

56. CONVERSION OF AZIMUTH (ANGLES) FROM DEGREES, MINUTES, AND SECONDS TO MILS

Table IIIb, page 375, TM 6-230 (conversion of degrees, minutes, and seconds to mils) contains seven independent subtables.

a. The left table at the top gives value in mils for each 100° for angles from 100° to 300°.

b. The middle table at the top gives value in mils for each 10° for angles from 10° to 90°.

c. The right table at the top gives value in mils for each 1° for angles from 1° to 9°.

d. The left table (second row) gives value in mils for each 10' for angles from 10' to 50'.

e. The right table (second row) gives value in mils for each 1' for angles from 01' to 09'.

f. The left table (bottom row) gives value in mils for each 10'' for angles from 10'' to 50''.

g. The right table (bottom row) gives value in mils for each 1'' for angles from 01'' to 09''. An example of converting degrees, minutes, and seconds to mils (azimuth 342° 26'' 36'') follows.

300°	=	5,333.33₥
40°	=	711.11₥
2°	=	35.56₥
20'	=	5.93₥
6'	=	1.78₥
30''	=	0.15₥
6''	=	0.03₥
342° 26' 36''	=	6,087.89 mils (round off to 6,088 mils)

57. EXERCISES

Use TM 6-230. See paragraph 68 for solutions.

a. Convert the following angles in mils to angles in degrees, minutes, and seconds (table III):

(1) 562.

(2) 1836.

(3) 3866.2.

(4) 5824.6.

b. Convert the following angles in degrees, minutes, and seconds to angles in mils (table III):

(1) $36^\circ\ 28'\ 15''$.

(2) $175^\circ\ 15'\ 10''$.

(3) $206^\circ\ 08'\ 35''$.

(4) $310^\circ\ 10'\ 13''$.

Section X. CONVERSION FACTORS

58. GENERAL

Artillery computations involve various units of measurement, and conversion to other units of measurement may be necessary. The conversion factors listed below are used in survey and gunnery computations.

To convert	Multiply number by	or	Add to logarithm of number
Yards to meters	0.914402		9.961137-10
Meters to yards	1.093611		0.038863
Kilometers to miles	0.621370		9.793350-10
Miles to kilometers	1.609347		0.206650
Meters to feet	3.280833		0.515984

To convert	Multiply number by	or	Add to logarithm of number
Feet to meters	.304801		9.484016-10
Degrees to mils	17.777778		1.249877
Mils to degrees	0.056250		8.750123-10

59. EXERCISES

See paragraph 69 for solutions.

a. Convert the following (do not use logs):

(1) 5,562 yards to meters.

(2) 3,321 meters to yards.

(3) 420 feet to meters.

(4) 630 meters to feet.

(5) 3,462 mils to degrees.

(6) 276 degrees to mils.

b. Convert the following by using logs:

(1) 7,250 yards to meters.

(2) 2,140 meters to yards.

(3) 330 feet to meters.

(4) 477 meters to feet.

SECTION XI

SOLUTIONS TO EXERCISES

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60. SOLUTIONS TO PARAGRAPH 10

a.	(1) 22 5/6	(3) 39 3/8
	(2) 11 1/5	(4) 11 5/12
b.	(1) 11 3/8	(3) 13 3/4
	(2) 4 5/6	(4) 14 17/20
c.	(1) 7/20	(3) 5 1/10
	(2) 7 7/24	(4) 66
d.	(1) 1 2/3	(3) 1/4
	(2) 2 11/14	(4) 14
e.	(1) 5.1672	(3) 31.20
	(2) 10.0	(4) 31.40
f.	(1) 30	(3) .253
	(2) .5	(4) 15.6
g.	(1) .643	(3) 10.375
	(2) .793	(4) 31.571
h.	(1) 100	(3) 9/16
	(2) 576	(4) 1.44

61. SOLUTIONS TO PARAGRAPH 16

a. (1) $4 + 15 - 19 + 7 = 7$

(2) $17 + 91 - 115 + 7 = 0$

(3) $6(-10) + \frac{18}{3}(6 \times 4 - 7) - (\frac{-21}{-7}) = -60 + 6(24 - 7) - 3 =$

$-60 + 102 - 3 = 39$

(4) $(3 \times 2) + (4 \times 7) - \frac{6}{3}(-2) - 4(27) = 6 + 28 + 4 - 108 =$

$38 - 108 = -70$

b. (1)
$$2X + 17 = 35$$
$$2X = 18$$
$$X = 9$$

(2)
$$2X + 16Y = 170$$
$$2X = 170 - 16Y$$
$$X = 85 - 8Y$$

(3)
$$\frac{4.1}{22635} = \frac{1}{X}$$
$$X = \frac{22635}{4.1} = 5520.7$$

(4)
$$\frac{2X}{3} = 170$$
$$X = \frac{170 \times 7}{2} = 595$$

(5)
$$\frac{3X}{5} + \frac{2Y}{7} = 21$$
$$\frac{3X}{5} = 21 - \frac{2Y}{7}$$
$$\frac{3X}{5} = \frac{147 - 2Y}{7}$$
$$X = \frac{(147 - 2Y)}{7} \times \frac{5}{3}$$
$$X = \frac{735 - 10Y}{21}$$

(6)
$$\frac{4.5}{X} = \frac{1}{36159}$$
$$X = 4.5\,(36159) = 162715.5$$

(7)
$$\frac{3.9}{45678} = \frac{X}{6532}$$
$$X = \frac{3.9\,(6532)}{45678} = .5577 \text{ or } .56$$

(8)
$$\frac{2X + 5}{9} = \frac{6X - 2}{4}$$
$$4\,(2X + 5) = 9(6X - 2)$$
$$8X + 20 = 54X - 18$$
$$46X = 38$$
$$X = \frac{38}{46} = .826$$

62. SOLUTIONS TO PARAGRAPH 25

a. (1) Angle B = 3,200 - (1,150 + 1,450) = 600₥

(2) Angle D = angle B = 600₥

(3) Angle E = angle A = 1,150₥

b. (1) Angle 1 = 3,200 - angle 2 = 3,200 - 950 = 2,250₥

(2) Angle 3 = angle 4
angle 3 + angle 4 = angle 1

$$\text{angle } 3 = \frac{\text{angle } 1}{2} = \frac{2,250}{2} = 1,125₥$$

(3) Angle 4 = 1,125₥

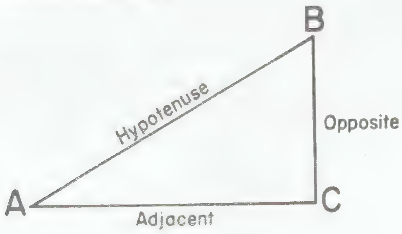
(4) Angle 5 = angle 2 = 950₥

c. Angle 1 + angle 2 + angle 3 + angle 4 + angle 5 = (n - 2) 3200
= 3(3200)

Angle 5 = 9600 - (angle 1 + angle 2 + angle 3 + angle 4)
= 9600 - (1700 + 1915 + 1320 + 4300)
= 9600 - 9235 = 365₥

63. SOLUTIONS TO PARAGRAPH 33

a.



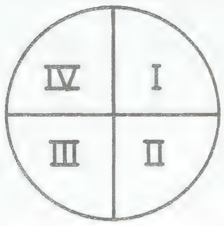
b. $$\text{Sin A} = \frac{\text{side opposite}}{\text{hypotenuse}}$$

$$\text{Cos A} = \frac{\text{side adjacent}}{\text{hypotenuse}}$$

$$\text{Tan A} = \frac{\text{side opposite}}{\text{side adjacent}}$$

$$\text{Cot A} = \frac{\text{side adjacent}}{\text{side opposite}}$$

c.



d. Angle, 20 mils; distance, 1,000 meters; width = 20 meters
Distance, 3,000 meters; width, 30 meters; angle = 10 mils
Angle, 5 mils; width, 20 meters; distance = 4,000 meters.

64. SOLUTIONS TO PARAGRAPH 36

a.

Length of b	= 5,000 meters
Angle A	= 700 mils
Angle B	= 1,500 mils
Angle C	= 1,000 mils
Side c	= 4,177.44 meters

$$c = \frac{b \sin C}{\sin B}$$

$$c = \frac{5,000 \sin 1,000}{\sin 1,500}$$

$$c = \frac{5,000 \times .83147}{.99519}$$

```
   .83147
     5000
4157.35000
```

```
             4177.44+
.99519 / 4157.3500000
         398076
          176590
           99519
           770710
           696633
            740770
            696633
             441370
             398076
              432940
              398076
               34864
```

b. Azimuth of line AB = 800 mils
Azimuth of line AC = 1,600 mils
Angle ACB = 1,100 mils

Solve for azimuth of line CB = 1,600 az line AC

$$
\begin{array}{rl}
1,600 & \text{az line AC} \\
+3,200 & \\
\hline
4,800 & \text{az line CA} \\
+1,100 & \text{angle ACB} \\
\hline
5,900 & \text{az line CB}
\end{array}
$$

Solve for angle B = 6,400

$$
\begin{array}{rl}
6,400 & \\
+\ 800 & \text{az of line AB} \\
\hline
7,200 & \\
-5,900 & \text{az of line CB} \\
\hline
1,300 & \text{angle B}
\end{array}
$$

65. SOLUTIONS TO PARAGRAPH 39

a. (1) 269.6 Fz 16.7
(2) 214.2 Fz 13.4
(3) 435.0 Fz 25.9
(4) 238.4 Fz 14.7

b. (1) R9 3,250, el 171.6
(2) R9 3,425, el 182.1
(3) R9 5,040, el 289.3

c. (1) 5,412
(2) 5,690
(3) 6,032

d. (1) 756012
(2) 370309
(3) 658974
(4) 050689

e. (1) 33205
(2) 53832
(3) 12404
(4) 14801

66. SOLUTIONS TO PARAGRAPH 48

a. (1) 9.321209-10
(2) 9.900662-10
(3) 9.912533-10
(4) 9.334524-10
(5) 2.123198
(8) 0.158610
(9) 4.387977
(10) 6.654177-10
(11) 0.04512
(12) 4.1840

(6) 8.755875-10 (13) 6654300

(7) 9.746907-10

b. (1) log 137.5 = 2.138303

+log 7.18 = 0.856124

log of the product = 2.994427

antilog (product) = 987.25

(2) log 2508 = 3.399328

+log sin 820₥ = 9.857847-10

log of the product = 13.257175-10

antilog (product) = 1807.92

c. (1) log 48.37 = 1.684576

-log 9.612 = 0.982814

log of the quotient = 0.701762

antilog (quotient) = 5.03224

(2) log 442 = 12.645422-10

-log tan 77.6 = 8.882703-10

log of the quotient = 3.762719

antilog (quotient) = 5790.52

67. SOLUTIONS TO PARAGRAPH 53

a. Determination of distance.

(1) $\tan M = \frac{dE}{dN}$

$\tan M = \frac{2593}{2510}$

55482	94370
58075	91860
+2593	-2510

log 2593 = 3.413803

-log 2510 = 3.399674

log tan M = 0.014129

M = 816.6m̸

Grid az AB = 3200m̸ - 816.6m̸ = 2383.4m̸

(2) S 816.6E

(3) $AB = \frac{dE}{\sin \text{bearing}}$

log dE = 13.413803-10

log sin 816.6 = 9.856449-10

log AB = 3.557354-10

AB = 3608.72 meters

b. Determination of coordinates.

(1) log dE = log 2150 + log sin 1095

log 2150 = 3.332438

+log sin 1095 = 9.944284-10

log dE = 13.276722-10

dE = 1891.13 meters

(2) log dN = log 2150 + log cos 1095

log 2150 = 3.332438

+log cos 1095 = 9.677351-10

log dN = 13.009789-10

dN = 1022.79

(3) 50450 96640

dE = -1891.13 dN = +1022.79

Cord = 48558.87 97662.79

68. SOLUTIONS TO PARAGRAPH 57

a. (1) 562 mils to degrees:

500 mils = 28° 07' 30.00"

60 mils = 3° 22' 30.00"

2 mils = 0° 06' 45.00"

562 mils = 31° 36' 45"

(2) 1836 mils to degrees:

1836 - 1000 = 836 mils

1000 mils = 56° 15' 00.00"

800 mils = 45° 00' 00.00"

30 mils = 1° 41' 15.00"

6 mils = 0° 20' 15.00"

1836 mils = 103° 16' 30"

(3) 3866.2 mils to degrees:

3866.2 - 3000 = 866.2 mils

3000 mils =	168° 45' 00.00"
800 mils =	45° 00' 00.00"
60 mils =	3° 22' 30.00"
6 mils =	0° 20' 15.00"
0.2 mils =	40.50"

3866.2 mils = 217° 28' 26"

(4) 5824.6 mils to degrees:

5824.6 - 5000 = 824.6 mils

5000 mils =	281° 15' 00.00"
800 mils =	45° 00' 00.00"
20 mils =	1° 07' 30.00"
4 mils =	0° 13' 30.00"
0.6 mils =	0° 02' 01.50"

5824.6 mils = 327° 38' 02"

b. (1) 36° 28' 15" to mils:

30° =	533.33 mils
6° =	106.67 mils
20' =	5.93 mils
08' =	2.37 mils
10" =	0.05 mils
05" =	0.02 mils

36° 28' 15" = 648.37 = 648 mils

(2) $175^{\circ}\ 15'\ 10''$ to mils:

$175^{\circ}\ 15'\ 10'' - 100 = 75^{\circ}\ 15'\ 10''$

100° = 1777.78 mils

70° = 1244.44 mils

5° = 88.89 mils

$10'$ = 2.96 mils

$05'$ = 1.48 mils

$10''$ = 0.05 mils

$175^{\circ}\ 15'\ 10''$ = 3115.60 = 3116 mils

(3) $206^{\circ}\ 08'\ 35''$ to mils:

$206^{\circ} - 200^{\circ} = 6^{\circ}$

200° = 3555.56 mils

6° = 106.67 mils

$08'$ = 2.37 mils

$30''$ = 0.15 mils

$05''$ = 0.02 mils

$206^{\circ}\ 08'\ 35''$ = 3664.77 = 3665 mils

(4) $310^{\circ}\ 10'\ 13''$ to mils:

$310^{\circ} - 300^{\circ} = 10^{\circ}$

300° = 5333.33 mils

10° = 177.78 mils

$10'$ = 2.96 mils

$10''$ = 0.05 mils

$03'$ = 0.01 mils

$310^{\circ}\ 10'\ 13''$ = 5514.13 = 5514 mils

69. SOLUTIONS TO PARAGRAPH 59

a. (1) 5,562 yards to meters:

```
     0.914402
         5562
      1828804
     5486412
    4572010
   4572010
  5085.903924 = 5,085.9 meters
```

(2) 3,321 meters to yards:

```
     1.093611
         3321
      1093611
     2187222
    3280833
   3280833
  3631.882131 = 3,631.9 yards
```

(3) 420 feet to meters:

```
    0.304801
         420
     6096020
   1219204
   128.016420 = 128.02 meters
```

(4) 630 meters to feet:

```
    3.280833
         630
    98424990
  19684998
  2066.924790 = 2,066.9 feet
```

(5) 3,462 mils to degrees:

```
     0.056250
         3462
       112500
      337500
     225000
    168750
    194.737500 = 194.74 degrees
```

(6) 276 degrees to mils:

```
  17.777778
        276
  106666668
 124444446
 35555556
4906.666728 = 4,906.7 mils
```

b. (1) 7,250 yards to meters:

```
  3.860338
  9.961137-10
 13.821475-10 = 6,629.4 meters
```

(2) 2,140 meters to yards:

```
 3.330414
 0.038863
 3.369277 = 2,340.3 yards
```

(3) 330 feet to meters:

```
  2.518514
  9.484016-10
 12.002530-10 = 100.584
              ≐ 100.6 meters
```

(4) 477 meters to feet:

```
 2.678518
 0.515984
 3.194502 = 1,565.0 feet
```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